

# Mathematical Modelling in the Process of Operating the Channel Hydropower Plants

Several channel power plants are established in Slovakia. Most of them are located on river Vah and river Danube. The present-day Vah cascade of hydroelectric power plants was constructed over 60 years. Individual power plants and entire groups of power plants were designed and built with different hydraulic parameters such as sizes of channels, discharges through power plants, etc. Therefore, the hydraulic structures of the Vah cascade are complicated. The power plants of the Vah cascade are utilized to cover variable and unscheduled electric system loads in the Slovak electric power system; such as peaks and power failure reserves. The power plant Gabčíkovo on the Danube is the greatest channel power plant in Slovakia. Due to the fact that System of water works Gabčíkovo – Nagymaros has not been build up according to the original contract, the power plant Gabčíkovo currently works in through flow mode. This paper shows the possibility of application of 1-D hydraulic model in the area of channel power plants controlling on the cascade Vah and cascade Danube.

Key words: HYDRO POWER PLANT, WATER WORK, HYDRAULIC PARAMETERS, TERRAIN MEASUREMENTS



## I. INTRODUCTION

The Vah is the most important, and after the Danube the largest river in Slovakia. It springs out in a mountainous area between two ranges of mountains - the High Tatras and the Low Tatras. It is formed by the confluence of the Biely Váh and the Čierny Váh rivers near the village of Kráľova Lehota. Along its course, which covers more than 367 km, the difference in the Vah's altitude surpasses 556 m. The Vah river basin covers an area of 16,969 km<sup>2</sup>, which represents 34% of the total territory of Slovakia. From a hydropower viewpoint the Váh River is a significant supply of renewable power. The Vah River produces 3,505 GWh, which is 47.6% of the 7,361 GWh total primary and technically usable hydropower potential in Slovakia.

Recently, the Vah River has often changed its channel bed. As the valley of the Vah is quite narrow and has been densely settled from ancient times, the river has caused many troubles to the valley's inhabitants. Two large water reservoirs, the Liptovská Mara dam built in the upper stream of the Vah, and the Orava dam, built in the upper stream of the Orava River, which is the most powerful tributary to the Vah, both became the key to the equalizing flow rate fluctuations as well as controlling the floods in the river with the possibility of long-term flow regulation. Both water reservoirs are also record breakers in Slovakia - the Liptovská Mara water reservoir has the largest total capacity of 360 million m<sup>3</sup>, and the Orava water reservoir has the largest submerged area of 35 km<sup>2</sup>.

The water from the reservoirs is utilized for generating electricity by peaking hydro power plants (HPP) built directly at these dams. Next to them regulating reservoirs with flow hydro power plants were built the Bešeňová power plant next to the Liptovská Mara water reservoir, and the Tvrdošín power plant next to the Orava water reservoir. Beside protection against floods, the main purpose of such a hydropower system is to provide higher water flow rates for all downstream power plants of the Vah cascade, which is very important in the increasing of the electrical energy production.

However, dense settlement, important industry and developed agriculture became limiting factors for the utilization of the water power of the Vah River, because it was not economically acceptable to draw relatively large areas which would otherwise be essential for the construction of a cascade composed of larger valley reservoirs. The utilization of the water power on the Vah is therefore resolved by the Vah cascade - a system of hydropower plants on side channels.

The construction of the Vah cascade of hydropower plants was started around 1930. The first power plant, Ladce, was built and put into operation in 1936. Due to their dimensions the hydro power plants and side channels built during the first years of the construction of the Vah cascade mostly permitted only semi-peak operation. Step by step, the opinions on the function of power stations and their position in the hydropower system have changed. Therefore, the larger-sized channels and power plants with larger absorption capacity of turbines installed were built in recent years which, together with the water reservoirs in the Vah valley, now make peak operation possible.

The large-scale construction of the Vah cascade was promoted after the Second World War. At that time not only separate hydropower plants were built, but also whole groups of them. This had a great effect upon the whole system of the Vah cascade.

At present the hydropower plants of the Vah cascade are divided into four groups (from top to bottom, see Fig. 1):

1. Krpeľany - Sučany - Lipovec
2. Hričov - Mikšová - Považská Bystrica
3. Ladce - Ilava - Dubnica - Trenčín
4. Kostolná - Nové Mesto - Horná Streda
5. Madunice – Sereď (Madunice channel hydro power plant was built on the downstream of the Vah River. After the construction of the Sereď water

works, they should both form the fifth group of hydropower plants on the Vah cascade)

At present the hydropower plants of the Vah cascade have a peak operation, which is provided by a hydropower dispatch centre in Trenčín. This enterprise belongs to the Slovenské elektrárne inc., which is the greatest producer of electric power in Slovakia.

The operation of the Vah cascade is optimised by means of computation with respect to the needs of the electric power system, the total water capacity in the reservoirs and the current water flow conditions in the Vah River. The processes of putting the hydropower plants into operation and their shutting down are fully automated. Hydrodynamic models for the hydropower dispatch centre have been developed for the provision of hydraulic links between separate hydropower plants.

This article is aimed at explaining the hydraulic links between the groups of channel hydraulic power plants (HPP) of the Vah cascade and the implementation of hydrodynamic models in the management of operations by the dispatch centre.

Power plant Gabčíkovo on the Danube is the greatest channel power plant in Slovakia (see Fig. 4). The installed capacity 720 MW was defined with respect to the originally proposed peak operation. The maximum flow rate of the installed turbines (4000 m<sup>3</sup>s<sup>-1</sup>, by overloading 5000 m<sup>3</sup>s<sup>-1</sup>) satisfies the peak operation of the power plant. According to the original contract, the incomplete construction of System of hydraulic structures Gabčíkovo – Nagymaros does not allow the originally planned peak operation of the hydraulic structure Gabčíkovo. This part is currently in through flow mode. Incomplete is mainly the construction of balancing reservoir of stage Nagymaros.

The channel water plants on the Vah cascade and the channel water plant Gabčíkovo on the Danube are controlled by the control system of Trenčín's company Vodne elektrarne. The aim of this paper is the understanding of the application of hydrodynamic models in the area of operation controlling of channel power plants mentioned above and hydraulic links within the group of the channel power plants on the Vah cascade.

## II. HYDRAULIC LINKS WITHIN THE GROUP OF THE CHANNEL HYDRO POWER PLANTS

The instantaneous flow rate in the Vah River is usually lower than the total absorption capacity of the turbines of the group of the channel HPP. The full turbine power can be achieved only by water diversion from the supply capacity of a reservoir before the entry of a side channel. As far as the Vah River is concerned, the HPP groups are part of the cascade; therefore, the flow rate conditions can also be affected by appropriate regulation of the upper-stream hydro power plants.

Since the flow rate conditions in this case are regulated, these groups of channel HPP are called regulation HPP. The profile of each of the hydro power plants always has equalled the regulated flow rate in the side channel, which is processed at the same time or gradually by the power plants, and the whole group is operated according to the first HPP on the side channel. This first HPP is also called the pilot HPP of the group, and the operation is called the tandem operation.

When the HPP group is put into tandem operation, the unsteady motion of the water is achieved in the side channel. The same situation is achieved after putting the HPP group out of operation or the change of flow rate during operations. The water motion in the channel

becomes steady after the sufficient long-term operation of a constant flow rate. Considering the peak character of the operation of the HPP group, the stabilization need not be achieved during short peaks.

The hydropower system of the channel HPP on the Vah River consists of three (or four) hydropower plants on the same side channel. On the upper stream and the down stream it is bounded by reservoirs. From a physical point of view, each hydropower plant represents a singular point. At this point it is possible to define the hydraulic parameters of the flow as well as its behaviour over time. In dealing with the hydraulic conditions and the hydraulic links of the group of the channel HPP, it is not necessary to consider the whole group at one time. We can divide the side channel into separate sections between two hydro power plants (interchannels). After the separation of the system, the singular points - HPP - can be considered as peripheral profiles, and the well-known behaviour over time of the hydraulic parameters in these profiles as the peripheral flow conditions in the separate interchannels.

For completeness' sake we want to note that the general hydropower system of the group of the channel HPP may also contain a supply channel leading from the reservoir to the first HPP in the group and a tail channel leading from the last HPP in the group to the reservoir behind the group of the HPP. In this case the inflow and outflow profiles of the whole group of the HPP are also singular points in which the system can be separated. The number of the sections (interchannels) will increase - the first reservoir and the last reservoir in the group.

## III. HYDRODYNAMIC MODELS

There are two basic types of unsteady flow in derivation channels:

1. Shock waves - caused by sudden changes in flow in the peripheral profiles. They represent the breakdown case of operation.
2. Shock-less waves - caused by gradual changes in flow in the peripheral profiles (HPP). They represent the smooth peak operation of HPP.

The unsteady flow is characterized by the change over time of the flow and the change in the water level in the separate profiles of the open channel. In the gradual change of these quantities (shock-less waves), the physical substance can be expressed by means of a primary system of modified Saint-Venant partial differential equations as follows:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} - q_\ell = 0$$

$$\frac{\partial (QV)}{\partial x} + \frac{\partial Q}{\partial t} + gA \frac{\partial h}{\partial x} = gA \left( -i_e \right) q_\ell v_\ell$$

where

$Q$	water discharge [m <sup>3</sup> s <sup>-1</sup> ]
$A$	flow area [m <sup>2</sup> ]
$q_\ell$	density of lateral side inflow or outlet [m <sup>3</sup> s <sup>-1</sup> ]
$x$	length coordinate [m]
$t$	time [s]
$V$	average section velocity [ms <sup>-1</sup> ]
$h$	water level [m]
$g$	gravitation acceleration
$\beta$	correction factor reflecting the influence of non-uniform velocity distribution
$i_0$	bottom gradient

$i_e$	power line gradient
$v_l$	velocity component of side inflow or outlet [ms <sup>-1</sup> ]

For the simulation of an unsteady flow in open channels (in this case, in interchannels), mathematical modelling is used, which takes into account:

- the mathematical interpretation of the physical fact of the matter (water flow in a channel)
- the selection of the appropriate numerical method for the solution of the system of equations so that the solution would be effective and stable
- the creation of a mathematical model of the environment (the system of channels) in which the examined hydraulic effect takes place. The model is defined by the set of discrete values which set up the geometric and hydraulic properties of this system in separate cross-sections
- the development of a program for computer calculation
- verification and calibration of model (see Fig. 2 and Fig. 3 for HPP on river Vah)

The results of the calculations of the unsteady flow in the group of the channel HPP have broad application, which can be divided into two basic areas:

1. The development of the principles of manipulation with the group of the HPP
2. Actual operation (see from Fig.5 to Fig. 8 – abrupt dropout HPP Gabčíkovo)

The first area represents calculations first and foremost in the stage of pre-project and project preparation or testing operations which are essential for the development of a manipulation order. The calculations particularly take into account:

- the extent of the admissible fluctuation of water levels during peak operation. First and foremost, the limited water levels are defined with respect to the methods and variability of operations of the HPP group, depending on the needs of the power system
- more precise operational water levels, so that the upper edges of the channel sealing are not over-flown
- operating provisions, which would enable the entire water works to fulfil its function with confidence
- the technical provisions, as the case may be, on the channels and the HPP (in older ones, for instance, their renewal and an increase in capacity)

The other area means the application in operation. The hydrodynamic models serve for effective operations.

Through them, a dispatcher can calculate the various possible variants of operation.

#### IV. CONCLUSION

Nowadays there exist hydrodynamic models for all the Vah cascade power plant groups and the hydraulic structure Gabčíkovo. These models are used by the operator – Slovenske elektrarne a.s., Vodne elektrarne o.z. located in Trenčín.

The improvement of planning and management of operations can be achieved by the exact definition of hydraulic links of the group of the channel HPP and the computation of the hydrodynamic models. This results in more reliable operations with the preservation of the regulatory functions of the HPP and the higher production of electric power.

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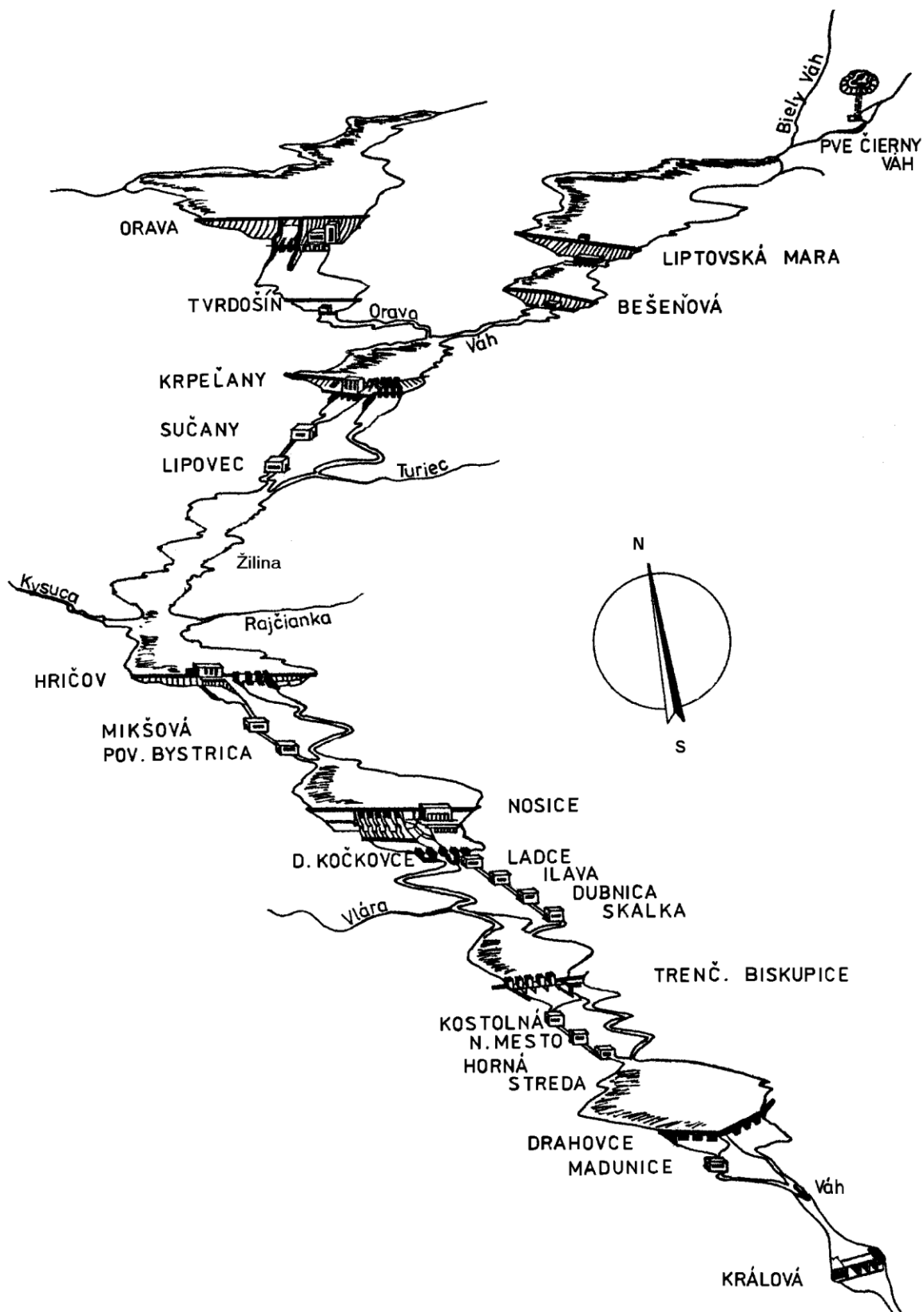


Fig.1 Váh cascade of hydropower plants

### Interchannel Hričov - Mikšová 16.11.2000

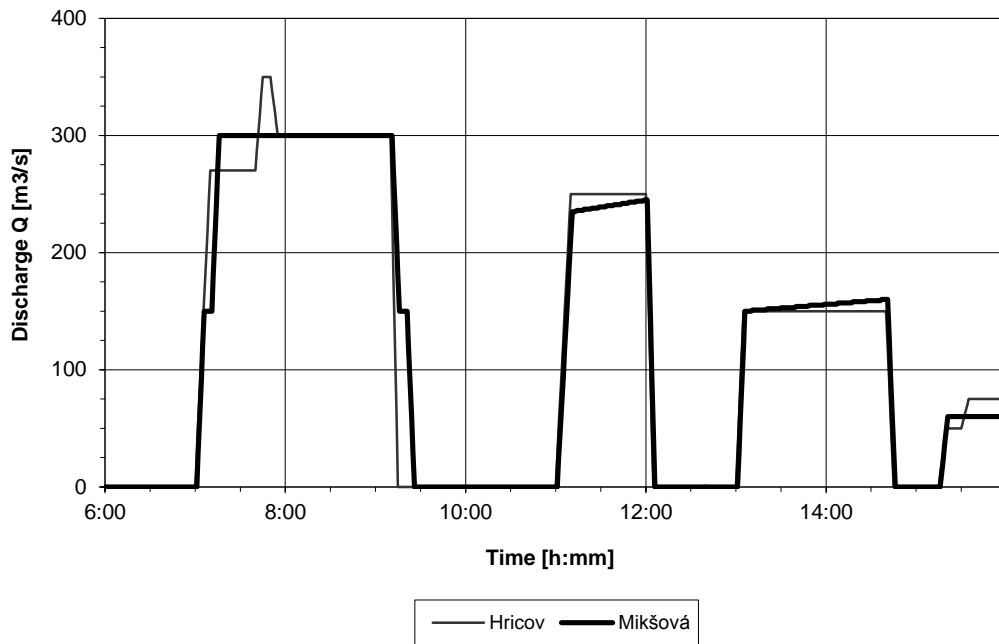


Fig. 2 Time manipulation of discharge

### Interchannel Hričov - Mikšová 16.11.2000

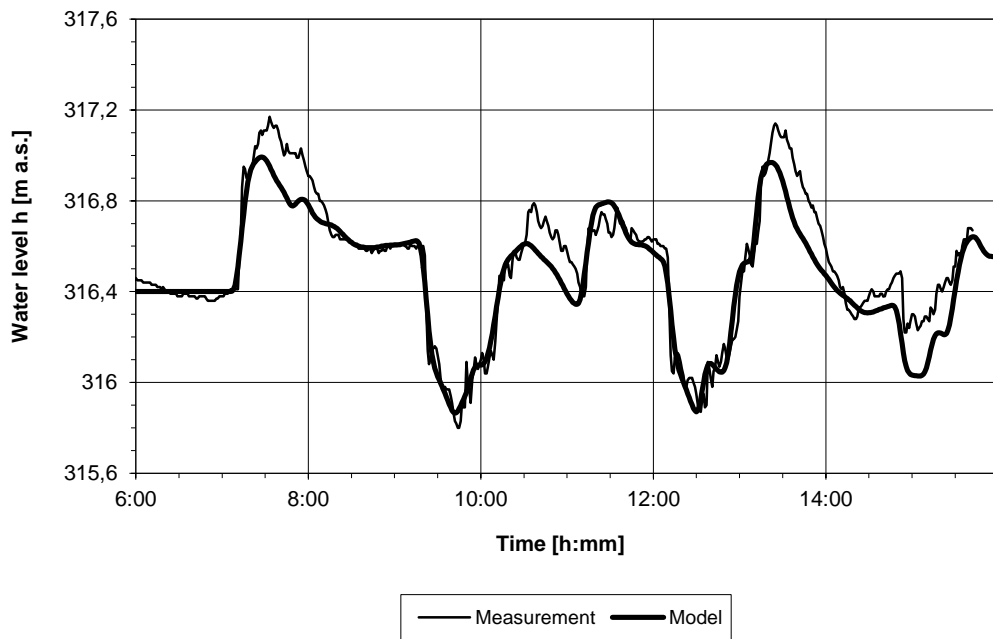


Fig. 3 Comparing water level measurements to results from model

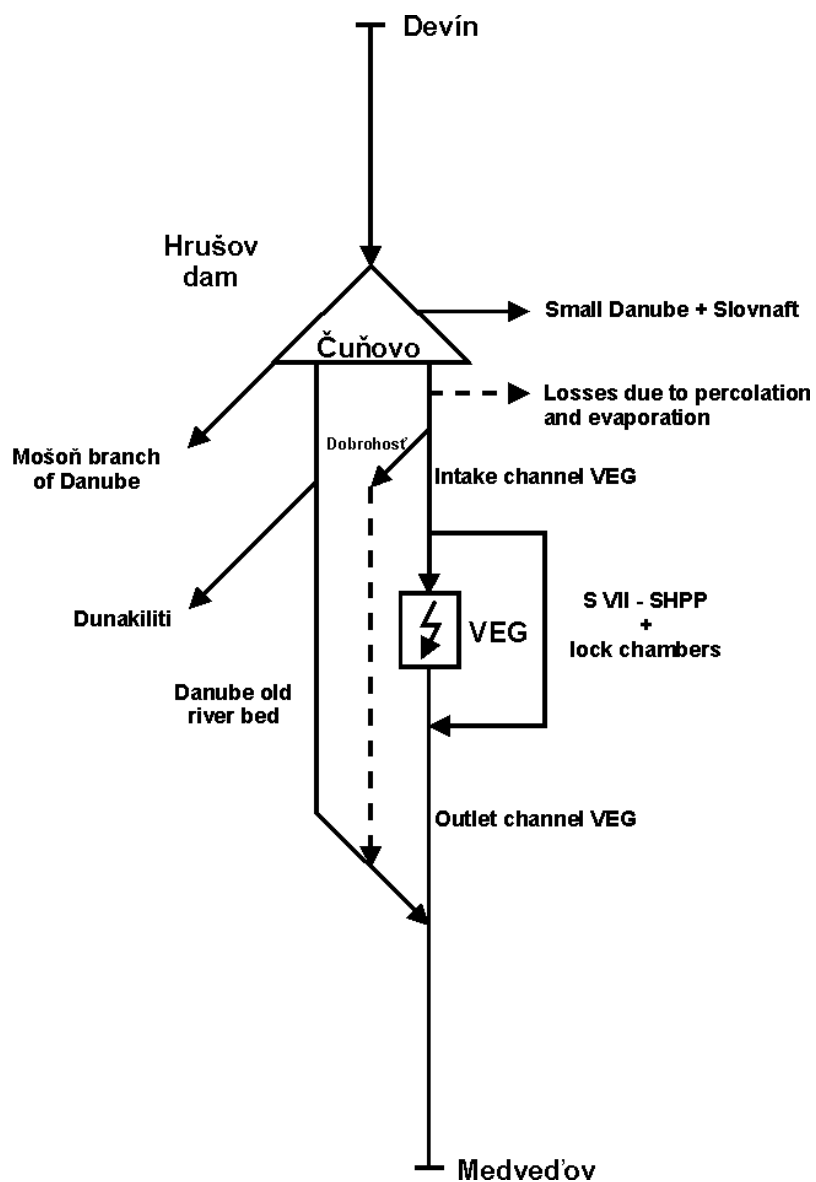


Fig. 4 Scheme of water work Gabčíkovo

### Hydroelectric power plant Gabčíkovo

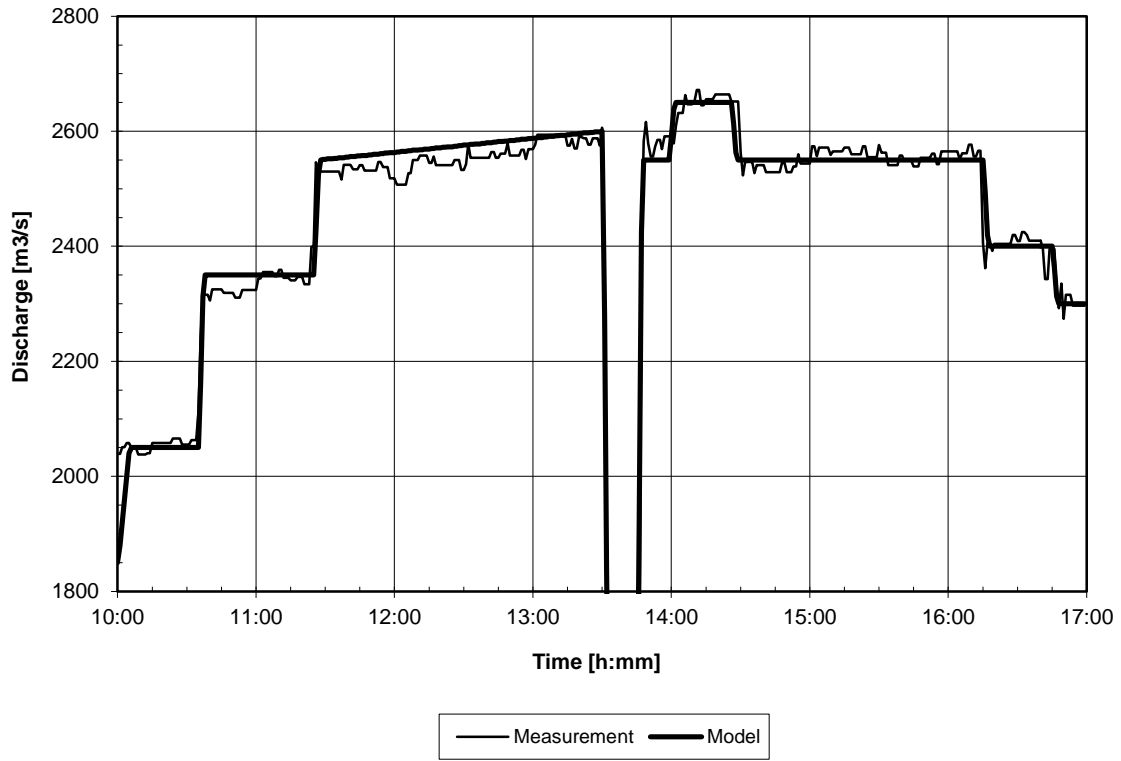


Fig. 5 Time manipulation of discharge

### Cross section at Ferry-boat Intake channel

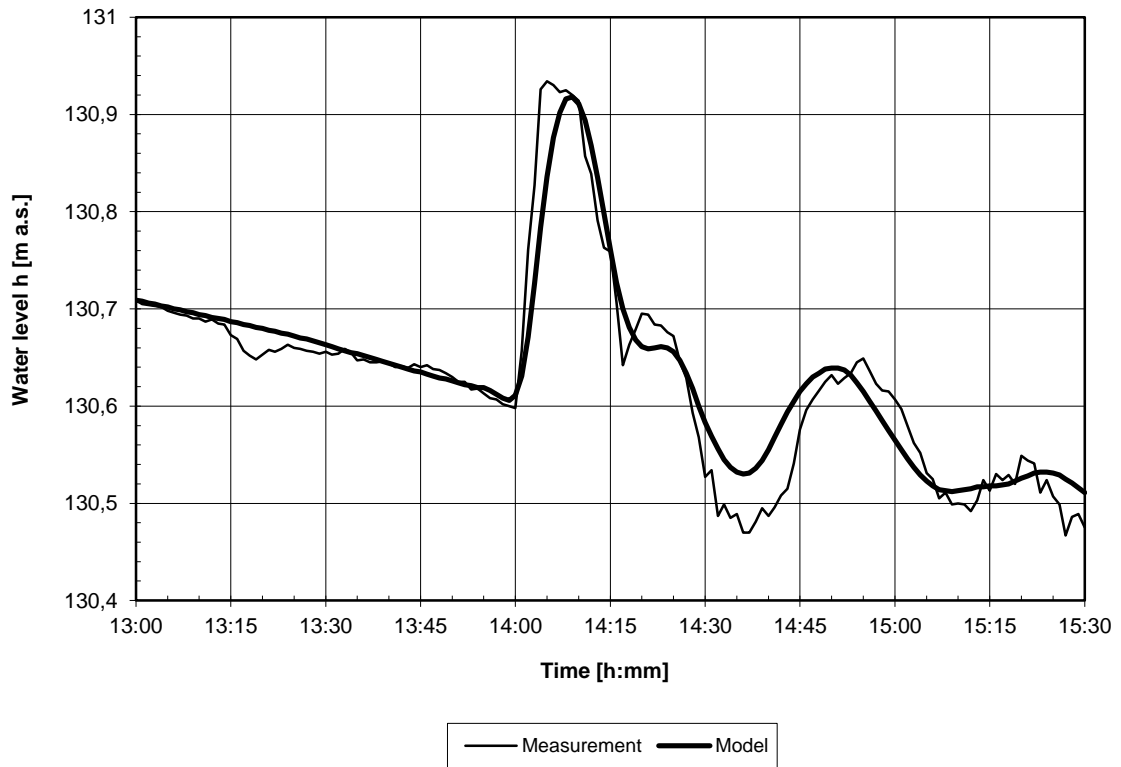


Fig. 6 Comparing water level measurements to results from model

### Cross section above HPP Gabčíkovo

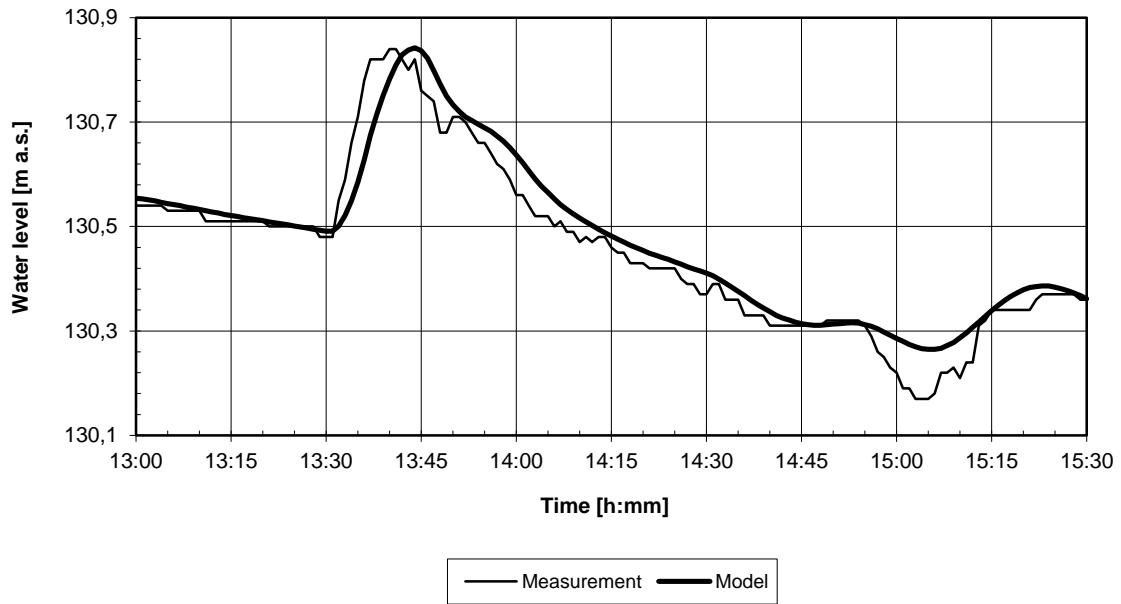


Fig. 7 Comparing water level measurements to results from model

### Cross section under HPP Gabčíkovo

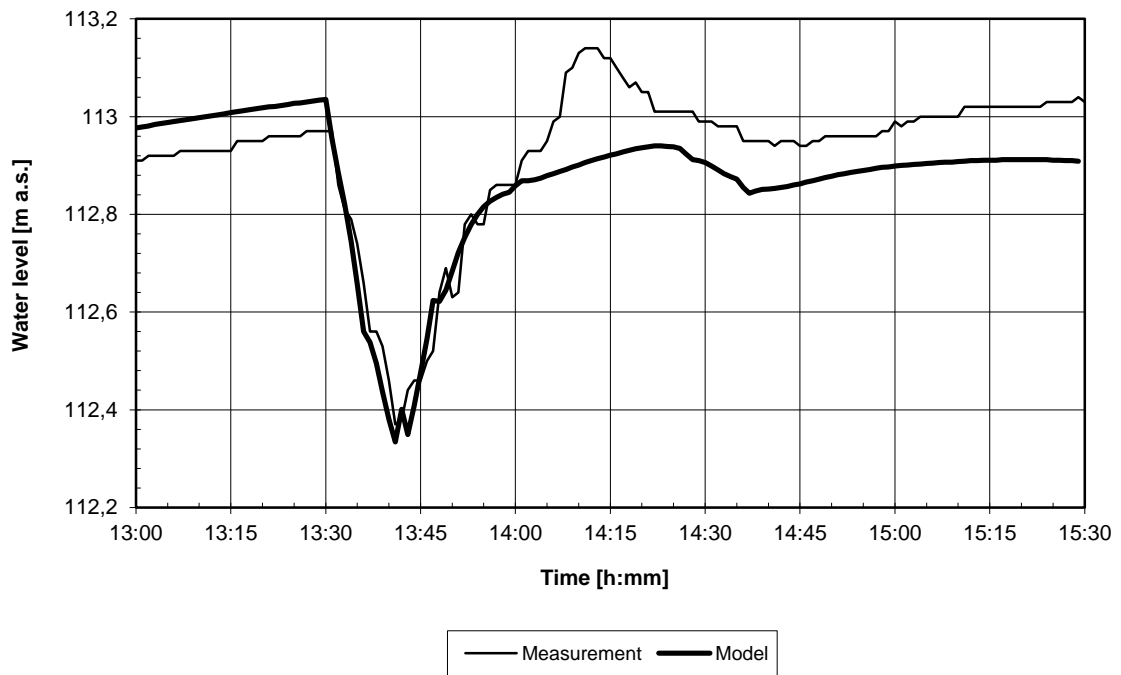


Fig. 8 Comparing water level measurements to results from model