

# Mathematical Model of the Hydropower Plant Madunice

The water work Drahovce - Madunice is the simplest part of Vah hydroelectric system from the point of view of mathematical modelling. Modelling methods verified on other channel hydro power plants are easily applicable on this hydroelectric system. These methods are based on decomposition of modelling system. The system is decomposed to separate sections with own defined boundary conditions. At the same time we can verify complex methods to obtain more realistic description of interaction between natural Vah riverbed (together with water reservoirs) and artificial channels of water power plants.

Key words: Hydro Power Plant, Water Work, Mathematical Model



## 1. INTRODUCTION

The water work Drahovce - Madunice is the simplest part of Vah hydroelectric system from the point of view of mathematical modelling (Fig.1). Modelling methods verified on other channel hydro power plants (Kamensky a kol.,1998) are easily applicable on this hydroelectric system. These methods are based on decomposition of modelling system. The system is decomposed to separate sections with own defined boundary conditions. At the same time we can verify complex methods to obtain more realistic description of interaction between natural Vah riverbed (together with water reservoirs) and artificial channels of water power plants (Dusicka, Kveton, 2001). The results from the modelling of the hydro power plant (HPP) Madunice will be applied to the other parts of Vah hydroelectric system. The goal of this application is to get a complex hydro dynamical model of the catchments of the river Vah.

The presented paper describes the hydro dynamical model (HDM) of HPP Madunice as a channel power plant model with an upper boundary condition water reservoir and an down boundary condition water jump over a small stone dam at confluence of outlet channel from HPP and the river Vah.

Solution of the mathematical model HPP Madunice is based on results from hydro dynamical modelling of water level regime in channels HPPs controlled by company Vodne elektrarne Trencin.. The water level regime in the intake and the outlet channels (as non permanent appearance by start and termination of working channel HPPs) belongs to important control characteristics of the channel HPPs. It is not only important from the energetic point of view, but mainly from the point of view of safety.

The unsteady flow is characterized by the change over time of the flow and the change in the water level in the separate profiles of the open channel. In the gradual change of these quantities (shock-less

waves), the physical substance can be expressed by means of a primary system of modified Saint-Venant partial differential equations as follows:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} - q_l = 0$$

$$\frac{\partial(QV)}{\partial x} + \frac{\partial Q}{\partial t} + gA \frac{\partial h}{\partial x} = gA(-i_e) - q_l v_l$$

where

$Q$	water discharge [ $\text{m}^3\text{s}^{-1}$ ]
$A$	flow area [ $\text{m}^2$ ]
$q_l$	density of lateral side inflow or outflow [ $\text{m}^3\text{s}^{-1}$ ]
$x$	length coordinate [m]
$t$	time [s]
$V$	average section velocity [ $\text{ms}^{-1}$ ]
$h$	water level [m]
$g$	gravitation acceleration
$\beta$	correction factor reflecting the influence of non-uniform velocity distribution
$i_0$	bottom gradient
$i_e$	power line gradient
$v_l$	velocity component of side inflow or outflow [ $\text{ms}^{-1}$ ]

The computer solution of the model of the water work (WW) Madunice – Drahovce was involved in the system of partial models solved by computer (Dusicka, Kveton, 2002). Together with the involving of described model in computer program, the system was extended with the following possibilities:

- boundary condition of the water reservoir, based on knowledge of the volume curve as function of the water level in the reservoir
- possibility to change the discharge through HPP by the manipulation on the installed hydraulic objects by setting time change of characteristic parameter

## 2. HYDRODYNAMIC MODEL OF WW DRAHOVCE – MADUNICE

The WW Drahovce Madunice was divided (from the point of view of modelling) to 2 sub-models :

- sub-model of the intake channel from the upper reservoir of HPP Madunice to Drahovce dam
- sub-model of the outlet channel from the confluence channel to the down reservoirs HPP Madunice

The cross sections (input topological data into model) were obtained from the project documentation of the WW. In comparison with documentation, the only difference found out was the upper level of the small stone dam at the confluence of the outlet channel and the river Vah. The measured level was 139, 10 m a.s. in comparison with projected level 139,40 m a.s.. This fact was confirmed by the measure of the water level under HPP by the permanent non-working stage. The other changes are expected mainly in the intake channel due to

the sediment accumulation from the reservoir Slnava. The volume curve of the reservoir Slnava (together with the outlet channel from HPP Horna Streda) and discharge curves through the installed hydraulic objects were taken over manipulation order of HPP. The measures on the WW Madunice – Drahovce were realised on 7-th September 2002. The discharge through HPP was controlled in order to the modified plan (see Fig.2). The different value of the discharge in experiment No.3 (the third peak in the discharge through HPP from 13:00 hour) between the measured discharge through HPP Madunice (only the discharge through turbines) in comparison with the discharge applied in model (total discharge through HPP) is due to the addition of the discharge through the navigation chamber.

## 2.1 HDM OF OUTLET CHANNEL HPP MADUNICE

Sub-model outlet channel is running with the boundary conditions:

- down boundary condition of type  $Q(h)$  – the discharge over the small stone dam in the depending of the water level
- upper boundary condition of type  $Q(t)$  – the time manipulation of the discharge through HPP

The down boundary condition  $Q(h)$  was not measured previously by user of HPP, therefore it was prepared on base of the water level measures in experiments No.1 and No.2 and it was verified in experiment 3.

## 2.2 HDM INTAKE CHANNEL OF HPP MADUNICE

Sub-model of the intake channel is running with the following boundary conditions :

- upper boundary condition of type  $h(t)$  – time function of the water level on the dam Drahovce , calculated from the volume curve of the reservoir Slnava as a function of the previous water level, inflows into reservoir and outflows from reservoir
- down boundary condition of type  $Q(t)$  – time manipulation of the discharge through HPP

In the part of the dynamic computing of the upper boundary condition the inflow from HPP Horna Streda has decisive position. The inflow to the reservoir Slnava in the cross section of the dam Drahovce was added to computing with a time delay, which was set up due to experimental fulfil of requirement to equal measured water level in the dam Drahovce and the computed water level in the same cross section.

## 3. COMPARISON OUTPUTS FROM HDM WITH MEASURES ON WW

The comparisons of the outputs from the model and the measured values on the outlet channel are on Fig. 3. There are compared the two most significant water levels (the water levels on the start and at the end of modelled section). The modelled channel is quiet short (4,6 km) and the agreement was achieved very well. Due to the relative short discharge peaks (max. 1,5 hour) there were not observed significant changes in the outlet channel depending on the rising water level in the natural river-bed of the river Vah under the small stone

dam. More complicated situation is by modelling of the intake channel where the stabilization of the water level needs longer time. The model achieved significantly better results for higher discharges (experiments No.1 and No.3). The biggest error was at the experiment No.2 where the experiment was started into an unsteady state with a relative small discharge (rolling reserve), which has minimal effect for the stabilisation of the water level.

On Fig.4 there are compared two results from modelling of the intake channel :

- time developing of the water level on the dam Drahovce – fulfilment of the upper boundary condition
- time developing of the water level on the HPP Madunice

The important using of the water wave regime is shown on Fig.5. The water wave (end of the experiment No.2) was used to clean trails of screens on the dam Drahovce .

## 4. CONCLUSIONS

The presented paper shows the possibilities of the water level regime models in channel HPP by energetic working. The integration of the natural river-bed data of the river Vah into the model would improve the quality of the model. After integration the model will be ready to use to model situations with raised discharges and flood discharges too.

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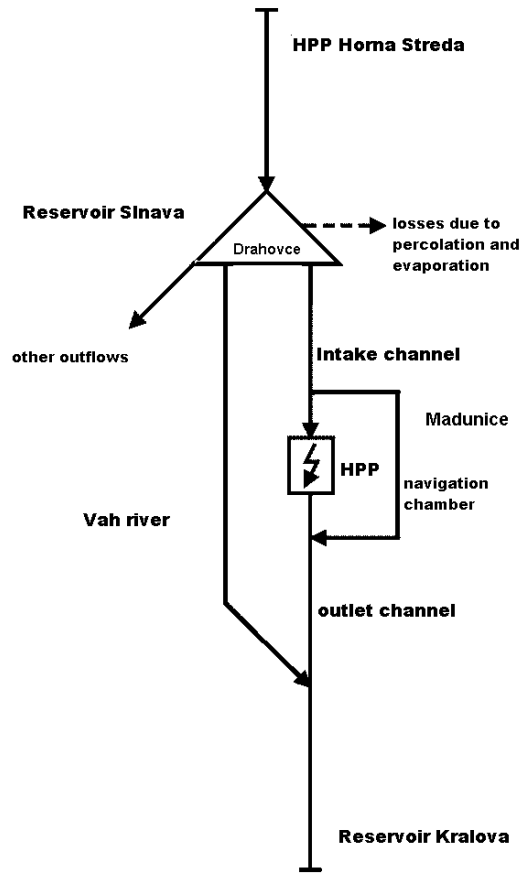


Fig.1 Scheme of water work Drahovce - Madunice

### Time developing of discharge

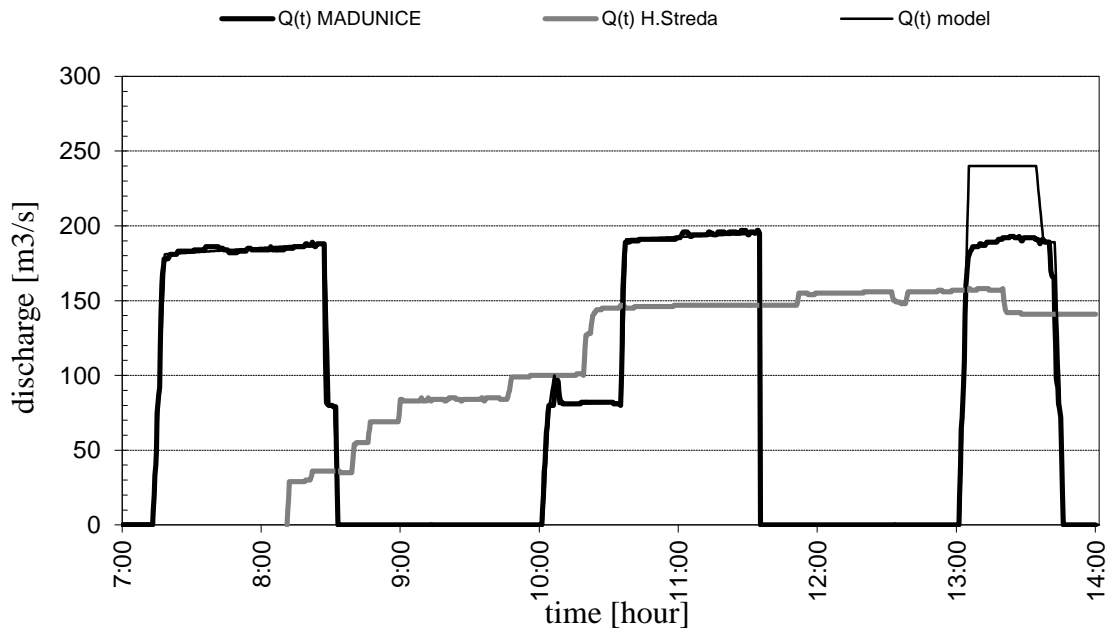


Fig. 2 Time manipulation of discharge HPP Madunice – experiments No.1 – No.3

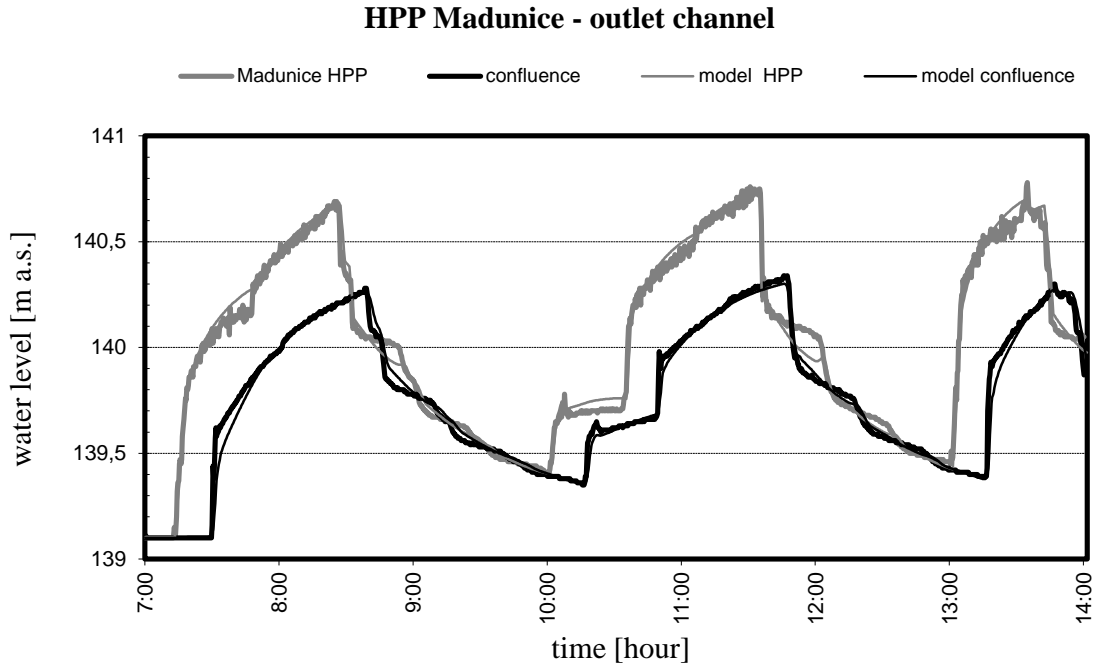


Fig. 3 Comparing water level measurements to results from model

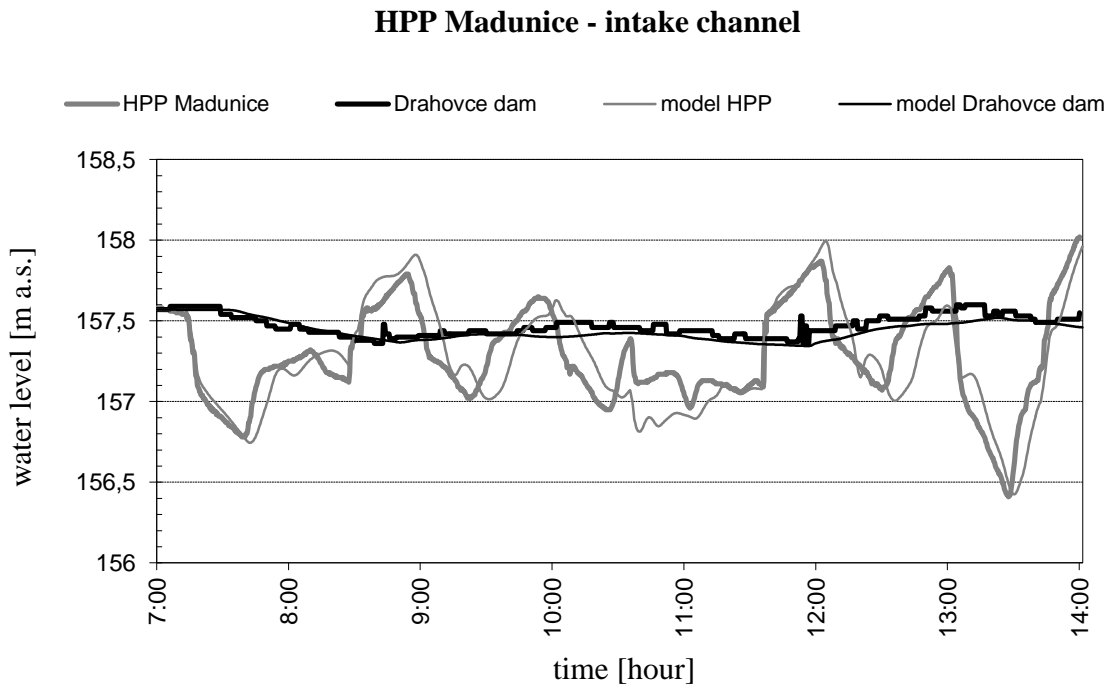


Fig. 4 Comparing water level measurements to results from model



Fig.5 Cleaning trails of screens using water wave