

**THE EFFECT OF WAVEGUIDE SHAPES ON THE MODAL
PROPERTIES AND AMPLIFICATION FACTORS**

Lenka ČIČMANCOVÁ, Milan NAĎ

ABSTRACT

Improving of machining process is one of the most important requirements that are expected from the machining methods. In many machining processes, the positive effects of ultrasound are applied. The performance of these machining systems and quality of machining process depends on the correct design of the individual system elements, mainly waveguide. For the correct functioning of the system, the waveguide must have the required dynamic properties - natural frequencies and amplification factors. The waveguide modal properties (natural frequencies, mode shapes) are determined by the numerical simulations using finite element method (FEM) design procedures. The effect of different waveguide shapes (conical, exponential, catenoidal) and their dimensions on modal properties and amplification factors are presented in this paper.

KEY WORDS

ultrasonic waveguide, modal properties, amplification factor, finite element method.

INTRODUCTION

Ultrasonic vibrations give considerable benefits for a variety of industrial applications. The remarkable advantages are achieved by the use of ultrasound in various machining processes. The different physical approaches of ultrasonic vibration energy applications can be presented in various machining processes. Two fundamental principles are used in ultrasonic machining processes, i.e. ultrasonic machining (USM) - ultrasonic transducer which is utilised indirectly to propel abrasive particles suspended in slurry at the work surface causing slow erosion and ultrasonic assisted machining (UAM) [1] - ultrasonic vibrations are transferred directly on the cutting tool and it can be applied to different machining technologies - turning, grinding, boring, milling and others. The repetitive high-frequency vibro-impact mode brings some unique properties and improvements into metal cutting process [1], [2], [3]. The electromechanical transducer of ultrasonic machining systems acts as the source of mechanical oscillations. The electro-mechanical ultrasonic transducers generate the vibration with frequency $f_{res} \approx 20$ kHz and more. The amplitude of the resulting ultrasonic vibrations is

Lenka ČIČMANCOVÁ, Milan NAĎ - Faculty of Materials Science and Technology, Slovak University of Technology, Paulínska 16, 917 24 Trnava, Slovak Republic, lenka.cicmancova@stuba.sk, milan.nad@stuba.sk

inadequate for realization of the cutting process. To overcome this problem, a waveguide focusing device is fitted onto the end of the transducer. The waveguide transfers the longitudinal ultrasonic waves from the transducer to end of the toe with attached the cutting tool. The cutting performance of ultrasonic machining equipment depends on the correct design of the waveguide. It amplifies the input amplitude of vibrations so that at the output end the amplitude is sufficiently large to required machining process [3]. Positive results in machining processes of hardly machinable materials, which usually could not be achieved by conventional machining processes are obtained by using UAM. The main benefits of ultrasound applications in machining [1], [2] are related to a reduction in cutting forces, improved machined surface quality, reduction wear and increasing life cycle of tool, etc.

The main aim is to present generally valid rules leading to the geometrical design of conical, exponential and catenoidal waveguide shape with required dynamical properties.

ULTRASONIC WAVEGUIDE DESIGN AND MATHEMATICAL MODEL

The main function of the waveguide is to amplify the amplitude of ultrasonic vibration of the tool cutting edge to the level required to the effective machining. The waveguide transfers the vibrational energy from the transducer towards to the tool interacting with workpiece. The manufacture and design of the waveguide require special attention because wrong manufactured waveguide will deteriorate performance of machining and can lead to the damage of the vibration system and cause considerable damage to the generator.

The most important aspect of the waveguide design is waveguide resonant frequency and the determination of the correct waveguide resonant wavelength. The required performance of a waveguide is assessed by an amplification factor [4]:

$$\mathfrak{G} = |A_1/A_0|, \quad \text{fundamental request: } \mathfrak{G} > 1, \quad (1)$$

where A_0 - amplitude of input end of horn, A_1 - amplitude of output end of horn.

Waveguides are made of isotropic material (Young's modulus E , density ρ , Poisson's number μ). The equation of motion of longitudinally vibrating ultrasonic waveguide with variable circular cross-section can be expressed in following form

$$\frac{\partial^2 u}{\partial t^2} - \frac{c_p^2}{S(x)} \frac{\partial}{\partial x} \left[S(x) \frac{\partial u}{\partial x} \right] = q(x, t), \quad (2)$$

where $u(x, t)$ - longitudinal displacement of cross-section in x-direction, $c_p = \sqrt{E/\rho}$ - velocity of propagation of the longitudinal waves in x-direction, $S(x) = S_0 f(x)$ - circular cross-section, $f(x)$ - function defining the change of the cross-section in longitudinal direction.

The solution of equation of motion for free vibrating (i.e. $q(x, t) = 0$) has the form $u(x, t) = U(x)e^{i\omega_0 t}$ and after substitution dimensionless quantities (Tab. 1) into (2), we can write

$$\frac{1}{S(\xi)} \frac{d}{d\xi} \left(S(\xi) \frac{d\bar{U}(\xi)}{d\xi} \right) + \beta^2 \bar{U}(\xi) = 0 \quad (3)$$

where $\beta = \omega_0 l_0 / c_p$ - frequency parameter, ω_0 - natural angular frequency, l_0 - length of waveguide.

THE DIMENSIONLESS QUANTITIES

Table 1

dimensionless coordinate	dimensionless longitudinal displacement	dimensionless ratio of diameter
$\xi = x/l_0 \quad \xi \in \langle 0; 1 \rangle$	$\bar{U}(\xi) = U(\xi)/l_0$	$\psi = d_1/d_0 \quad \psi \in \langle 0; 1 \rangle$

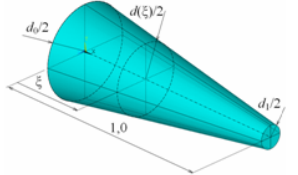
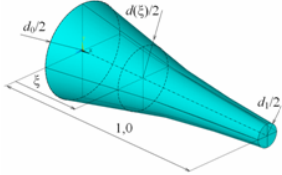
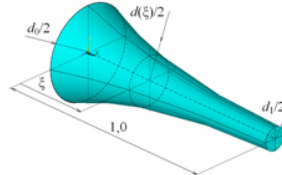
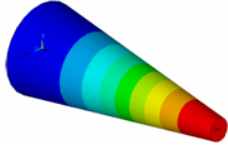
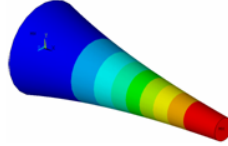
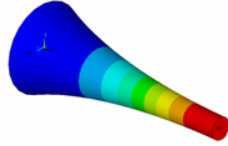
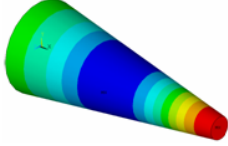
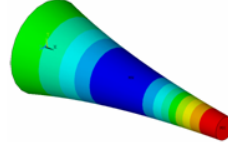
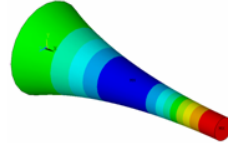
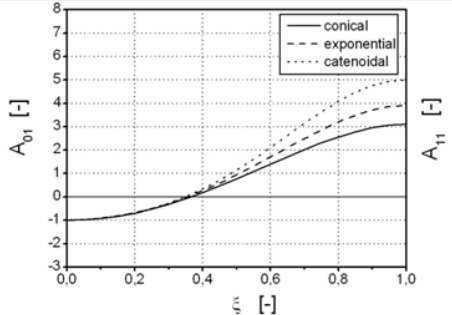
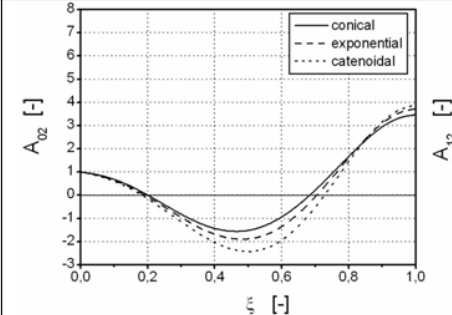
Generally, in machining processes only two vibrating mode shapes of waveguides are used, i.e. “half wave” shape and “wave” shape (Tab. 2). The most common shapes and their geometrical dimensionless parameters are shown in Tab. 2.

ANALYSIS AND RESULTS

The numerical simulation and determination of modal properties of considered waveguides shapes are carried out. The numerical analyses are performed for parameters defined in Tab. 2. The steel as a waveguide material is used to numerical simulation.

GEOMETRICAL PARAMETERS OF WAVEGUIDE SHAPES

Table 2

	CONICAL	EXPONENTIAL	CATENOIDAL	
Slenderness ratio $\bar{\delta}$: $\bar{\delta} = \frac{d_0}{l_0}; \quad (l_0 = 1,0)$				
Mode shapes	1 st „half wave”			
	2 nd „wave”			
Shape parameters	$d_0 = \bar{\delta}; \quad d(\xi) = d_0 f(\xi); \quad d_1 = \psi d_0 \Rightarrow$		$S(\xi) = S_0(f(\xi))^2$	
Shape function	$f(\xi) = 1 - \xi(1 - \psi)$	$f(\xi) = \psi^\xi$	$f(\xi) = \frac{\cosh[\kappa(1 - \xi)]}{\cosh(\kappa)}$ $\kappa = \ln\left(\frac{1 + \sqrt{1 - \psi^2}}{\psi}\right); \quad \psi \leq 1,0$	
Mode shapes: for dimensionless parameters: $\bar{\delta} = 0,4$ and $\psi = 0,25$ A_{0k} - input amplitude A_{1k} - output amplitude				

The dimensionless resonant frequencies for different geometrical shapes of waveguides can be expressed as

$$\theta_k = \frac{f_{0,k}^{shaped}}{f_{0,k}^{cylindrical}}, \quad \text{resp.} \quad f_{0,k}^{shaped} = \theta_k f_{0,k}^{cylindrical} \quad (4)$$

where $f_{0,k}^{shaped}$ - k^{th} natural frequency of shaped waveguide, $f_{0,k}^{cylindrical}$ - k^{th} natural frequency of cylindrical waveguide for corresponding slenderness ratio.

The dependencies of dimensionless frequencies θ_1 , resp. θ_2 on the slenderness ratio δ for the various ψ parameters are shown in Fig. 1÷Fig. 3 and the dependencies of the amplification factor ϑ_1 , resp. ϑ_2 on the ratio δ for various ψ are shown in Fig. 4÷Fig. 6.

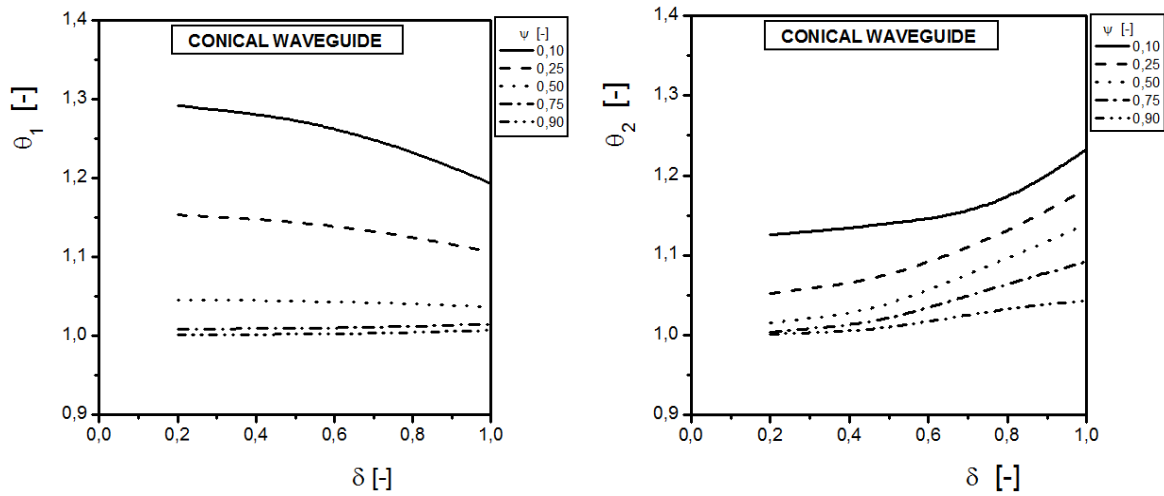


Fig. 1 Dependency of θ_1 , resp. θ_2 vs. slenderness ratio δ for conical waveguide shape

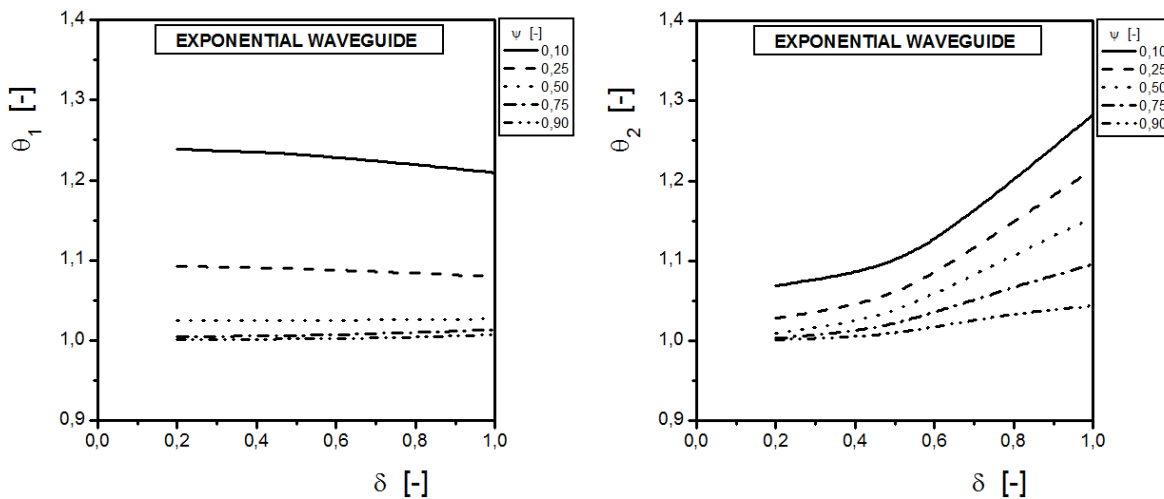


Fig. 2 Dependency of θ_1 , resp. θ_2 vs. slenderness ratio δ for exponential waveguide shape

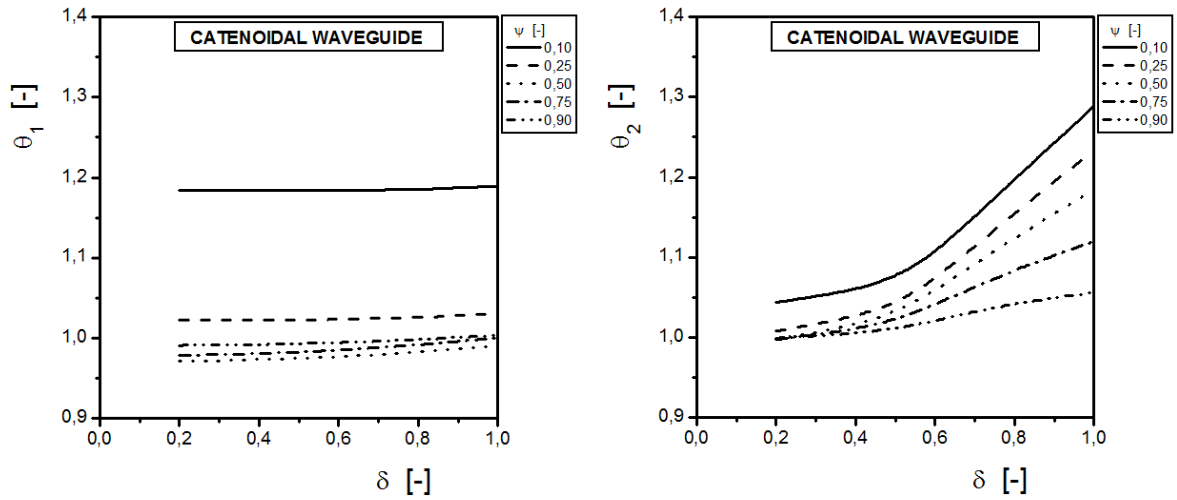


Fig. 3 Dependency of θ_1 , resp. θ_2 vs. slenderness ratio δ for catenoidal waveguide shape

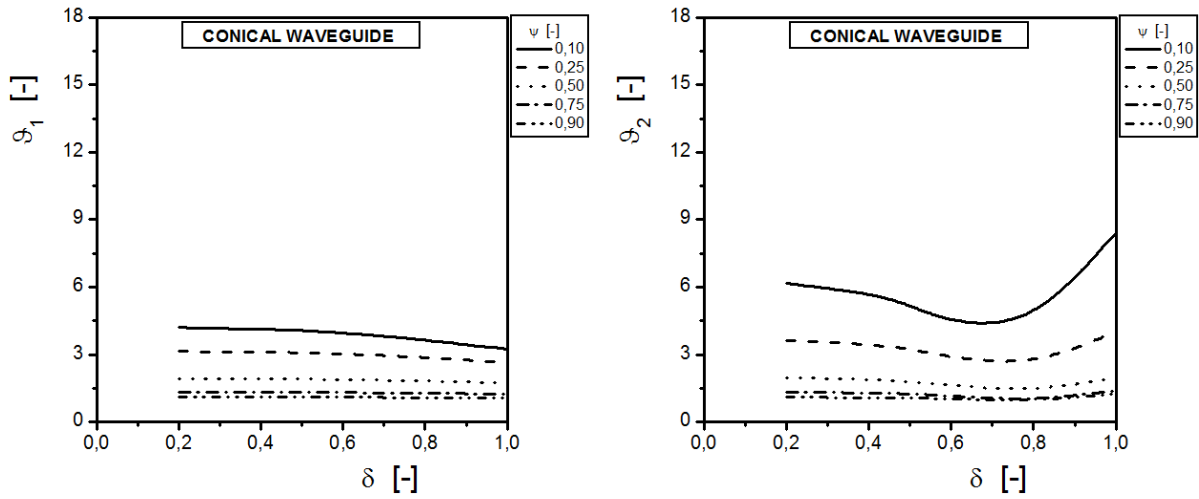


Fig. 4 Dependency of g_1 , resp. g_2 vs. slenderness ratio δ for conical waveguide shape

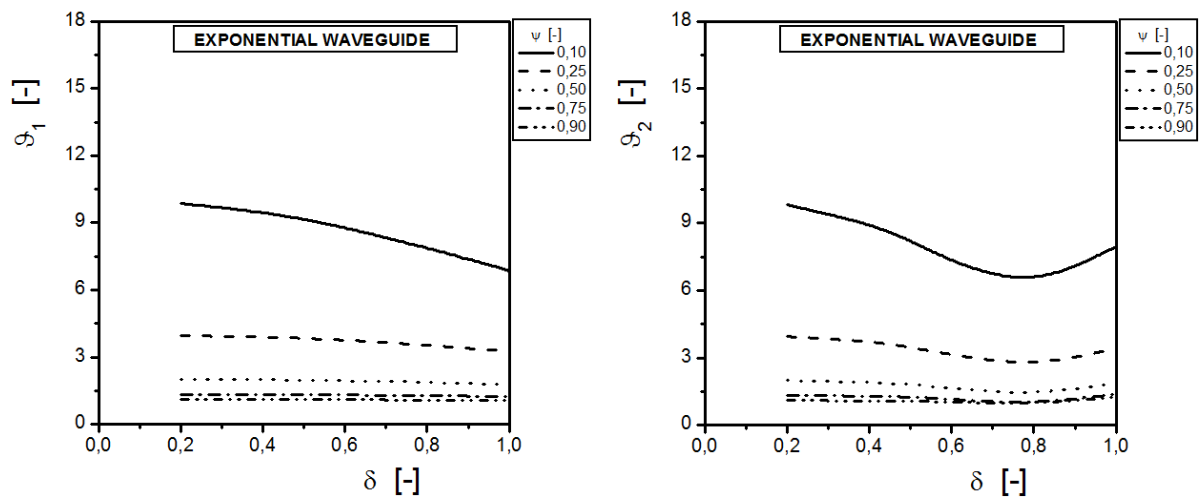


Fig. 5 Dependency of g_1 , resp. g_2 vs. slenderness ratio δ for exponential waveguide shape

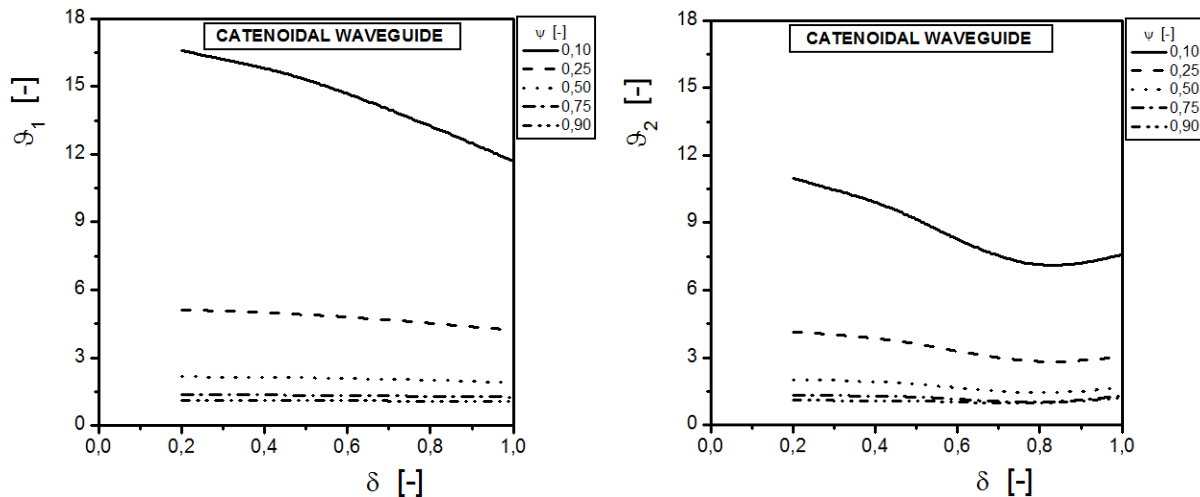


Fig. 6 Dependency of G_1 , resp. G_2 vs. slenderness ratio δ for catenoidal waveguide shape

CONCLUSION

The effect of the waveguide geometrical shape for ultrasonic manufacturing technologies is analysed in this paper. The different geometrical waveguide shapes were considered. The effects such as a slenderness ratio δ and ratio of output to input diameter on natural frequencies and amplification factor had been analysed.

Using Eq. (4), the required value of resonant frequency is determined. The fundamental requirements to the application of these waveguides for ultrasonic machining technologies are that the resonance frequency has to be more than 20 kHz and amplification factor is greater than 1,0.

ACKNOWLEDGMENT

The support of research project VEGA-1/0389/11 of Ministry of Education, Science, Research and Sport of the Slovak Republic is gratefully acknowledged.

REFERENCES

1. ASTASHEV, V.K., BABITSKY, V.I. 2007. *Ultrasonic processes and machines: Dynamics, control and applications*. New York: Springer Science+Business Media. ISBN 978-3-540-72060-7
2. BABITSKY, V.I., KALASHNIKOV, A.N., MEADOWS, A., WIJESUNDARA, A.A.H.P. Ultrasonically assisted turning of aviation materials. *Journal of Materials Processing Technology*, **132**(1-3), 157-167, 2003. ISSN 0924-0136.
3. NAĐ, M., ČIČMANCOVÁ, L. The effect of the shape parameters on modal properties of ultrasonic horn design for ultrasonic assisted machining. In: *Proceedings of the 8th International Conference of DAAAM Baltic Industrial Engineering*: Tallinn, Estonia 19-21 April 2012. Tallinn: Tallinn University of Technology, 57-62. ISBN 978-9949-23-265-9.
4. NAĐ, M., ČIČMANCOVÁ, L. Effect of shape parameters on the modal properties of stepped ultrasonic concentrator. *Applied Mechanics and Materials*, **309**, 43-49, 2013. ISBN 978-3-03785-636-9.