THE EFFECT OF INNER REINFORCING CORES ON NATURAL FREQUENCIES OF THE LATHE TOOL BODY

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ABSTRACT

The contribution is mainly focused on research and development of structural modification of machine tools, lathes in particular. The main aim of the modification is to change the modal properties (mode shapes, natural frequencies) of the lathe tool. The main objective of the contribution will be to formulate, mathematical analyse and evaluate the proposed methods and procedures for structural modifications of the tool, represented by beam body. A modification of modal properties by insertion of beam cores into beam body is studied in this paper. In this paper, the effect of material properties and geometrical parameters of reinforcing cores on natural frequencies of beam body is presented. The implementation will bring benefit on machine productivity, decreasing the machine tool wear and in many cases it will lead to better conditions in the cutting process.

KEY WORDS

lathe tool, modal properties, Euler-Bernoulli beam theory, vibration, reinforcement

INTRODUCTION

During the technological process, the various excitation effects affect the machine tools, for example lathe tools, drills, boring bars, etc. Cutting speeds, cutting forces, chip-making manner, the stiffness of MTW (machine-tool-workpiece) system are predominant effects affecting the dynamics of MTW system and also influencing the machining process (roughness of the machined surface, tool wear, tool or workpiece damage, noise generated by the machining process, etc.).

Therefore this paper is mainly focused on the analysis and modification of dynamic properties of the beam body. Dynamic properties are mainly represented by the modal properties (mode shapes, natural frequencies) that depend on the geometrical parameters and material properties of beam structure. An Euler-Bernoulli beam theory [1] is used as a simple model to illustrate the structural dynamic properties of a tool body in this paper. The Euler-Bernoulli beam model assumes that the deflection of the centerline is small and only transverse. While this theory assumes the presence of a transversal shear force, it neglects any

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shear deformation and the rotary inertia is also neglected. In the case of improper modal properties, the required modal properties of beam can be obtained by appropriate structural modifications of the beam body. In this paper, the modification of dynamic properties of beam is based on changes of material properties and geometrical parameters of beam cores.

THEORETICAL FORMULATION

The modal properties of beam with uniform double axes symmetric cross section are analysed. The beam cross section consists of a basic profile and reinforcements with longitudinally uniform cross section, which are embedded into beam body (Fig.1).
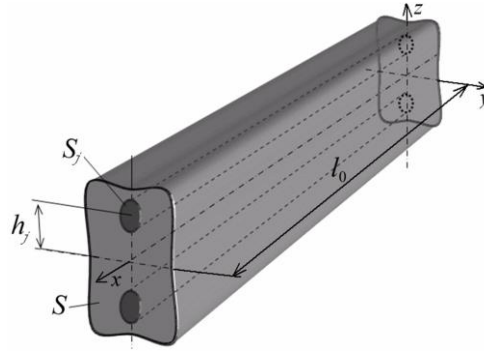

Fig. 1 *General model of beam structural element with reinforcing cores*

Mathematical model of beam body without reinforcing cores

The equation of motion for the free bending vibration of homogeneous Euler-Bernoulli beam with constant cross-section and without reinforcements [1] is in the form

$$EJ\frac{\partial^4 w(x,t)}{\partial x^4} + \rho S\frac{\partial^2 w(x,t)}{\partial t^2} = 0, \tag{1}$$

where S is the cross section area, EJ is the beam bending stiffness, J is the quadratic moment of the beam cross section, w is the beam deflection, ρ is the density and E is the Young´s modulus of beam material.

The equation (1) has supposed solution in the form $w(x,t) = W(x)T(t)$ and after introducing dimensionless parameters
• dimensionless deflection of beam $\quad \overline{W}(\overline{x}) = W(x)/l_0$, $\tag{2}$
• dimensionless coordinate $\quad\quad\quad \overline{x} = x/l_0$, for $\overline{x} \in \langle 0.0; 1.0 \rangle$, $\tag{3}$
is transformed into ordinary differential equation of the fourth order

$$\overline{W}^{IV}(\overline{x}) - \beta^4 \overline{W}(\overline{x}) = 0. \tag{4}$$

The solution of equation (4) has the form

$$\overline{W}(\overline{x}) = A\sin\beta\overline{x} + B\cos\beta\overline{x} + C\sinh\beta\overline{x} + D\cosh\beta\overline{x}, \tag{5}$$

where *A, B, C, D* are integration constants and β is the frequency parameter.

Generally, the boundary conditions for cantilever beam [3] are expressed by

$$\overline{W}(0) = \overline{W}'(0) = \overline{W}''(1) = \overline{W}'''(1) = 0. \tag{6}$$

Then, the natural angular frequency for the geometric parameters and material properties of the homogeneous beam can be expressed in the form

$$\omega_i = \left(\frac{\beta_i}{l_0}\right)^2 \sqrt{\frac{EJ}{\rho S}}, \tag{7}$$

where β_i is the frequency parameter determined from frequency equation (Tab.1).

Mathematical model of beam body with reinforcing cores

Next, we consider the beam body with uniform cross section of basic beam profile having inserted reinforcing cores with uniform circular cross section. The "reinforcing" core is not correct term. Reinforcing effect occurs only when the core structural parameters raise the stiffness properties of the basic beam. In formulating the mathematical model of the beam structure modified by reinforcing core, the following assumptions are considered:

- reinforcing cores are symmetrical with respect to both axes y, z,
- beam cross section of the is perpendicular to the neutral axis x,
- isotropic and homogeneous material properties of structural parts of the beam,
- beam cross section before and during deformation is assumed as planar,
- perfect adhesion is supposed at the interface of structural parts of the beam.

The equation of motion for free bending vibration of the modified beam has the form

$$EJ(1+\Delta_{EJ})\frac{\partial^4 w(x,t)}{\partial x^4} + \rho S(1+\Delta_{\rho S})\frac{\partial^2 w(x,t)}{\partial t^2} = 0 \tag{8}$$

where Δ_{EJ} and $\Delta_{\rho S}$ are dimensionless modification parameters.

The dimensionless parameters Δ_{EJ}, $\Delta_{\rho S}$ defining changes in structural parameters are:

- dimensionless mass modification $\qquad \Delta_{\rho S} = \sum_j (\delta_{\rho,j} - 1)\delta_{S,j} , \tag{9}$

- dimensionless stiffness modification

$$\Delta_{EJ} = \sum_j (\delta_{E,j} - 1)\delta_{J,j} = \sum_j (\delta_{E,j} - 1)\delta_{S,j}\delta_{\xi,j}, \tag{10}$$

- dimensionless density $\qquad \delta_{\rho,j} = \rho_j / \rho , \tag{11}$

- dimensionless Young's modulus $\qquad \delta_{E,j} = E_j / E , \tag{12}$

- dimensionless cross-section $\qquad \delta_{S,j} = S_j / S , \tag{13}$

- dimensionless quadratic moment $\qquad \delta_{J,j} = J_j / J , \tag{14}$

- dimensionless radius of gyration $\qquad \delta_{\xi,j} = (\xi_j^2 + h_j^2)/\xi^2$, where $\xi^2 = J/S \tag{15}$

where ρ_j - j^{th} core density, S_j - j^{th} core cross section area, J_j - j^{th} core quadratic moment of cross section, E_j - j^{th} core Young's modulus, and h_j - position of the jth core (Fig.1).

By using solution $w(x,t) = W(x)T(t)$ and the relations (2), (3) in equation (8), the equation (4) is valid and frequency parameter β of modified beam structure is expressed by

$$\beta_i = \sqrt[4]{\omega_{m,i}^2 l_0^4 \frac{\rho S(1 + \Delta_{\rho S})}{EJ(1 + \Delta_{EJ})}}, \tag{16}$$

where $\omega_{m,i}$ i^{th} natural angular frequency of modified beam structure. It can be defined by

$$\omega_{m,i} = \omega_i f_m(\Delta_{\rho S}, \Delta_{EJ}), \tag{17}$$

where $f_m(\Delta_{\rho S}, \Delta_{EJ}) = \sqrt{\dfrac{1 + \Delta_{EJ}}{1 + \Delta_{\rho S}}}$ - modification function. $\tag{18}$

The modification function $f_m(\Delta_{\rho S}, \Delta_{EJ})$ represents the structural changes within beam caused by changes in core structural parameters (density, core diameter and position of the both reinforcements to the neutral axis of the beam structure).

FREQUENCY EQUATIONS AND THEIR ROOTS FOR DIFFERENT BEAM
BOUNDARY CONDITIONS   Table 1

Case	Boundary conditions		Frequency equation	Roots		
	$\xi = 0$	$\xi = 1$		β_1	β_2	β_n
1	F	F	$\cosh(\beta)\cos(\beta) = 1$	4.73	7.85	$\frac{2n+1}{2}\pi$
2	SS	SS	$\sin(\beta) = 0$	π	2π	$n\pi$
3	C	C	$\cosh(\beta)\cos(\beta) = 1$	4.73	7.85	$\frac{2n+1}{2}\pi$
4	C	SS	$\tanh(\beta) = \tan(\beta)$	3.93	7.07	$\frac{4n+1}{4}\pi$
5	C	F	$\cosh(\beta)\cos(\beta) = -1$	1.88	4.69	$\frac{2n-1}{2}\pi$
6	F	SS	$\tanh(\beta) = \tan(\beta)$	3.93	7.07	$\frac{4n+1}{4}\pi$
F – free; SS – simply supported; C – clamped						

Modal properties (natural frequencies, mode shapes) of the beam structural element depend on the beam attachments to the foundation. The frequency equations and their roots for different boundary conditions are expressed in (Tab.1).

ANALYSIS AND RESULTS

The influence of changes in structural parameters of reinforcing cores on the modification function f_m is expressed by the dependency on mass modification $\Delta_{\rho S}$ or stiffness modification Δ_{EJ}. The dependencies of the function f_m on mass modification $\Delta_{\rho S}$, for stiffness modification parameters Δ_{EJ} are shown in Fig.2.
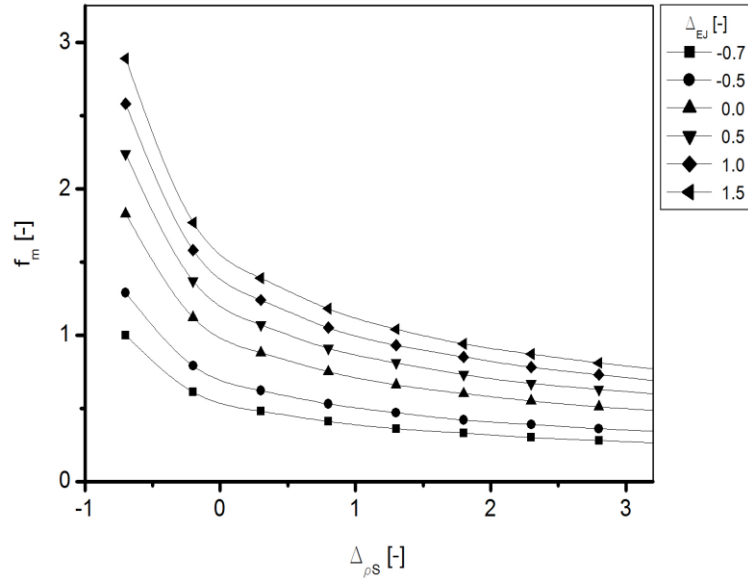
Fig. 2 *The dependency of f_m on $\Delta_{\rho S}$ for different flexural rigidity modification Δ_{EJ}*

The influence of stiffness modification Δ_{EJ} on the modification function f_m, for mass modification parameters $\Delta_{\rho S}$, is shown in Fig. 3. As it follows from the assessment of the above mentioned dependencies, the growth of the mass modification value $\Delta_{\rho S}$ causes the decrease of modification function and if the value of stiffness modification Δ_{EJ} is increasing, the modification function has an upward trend.
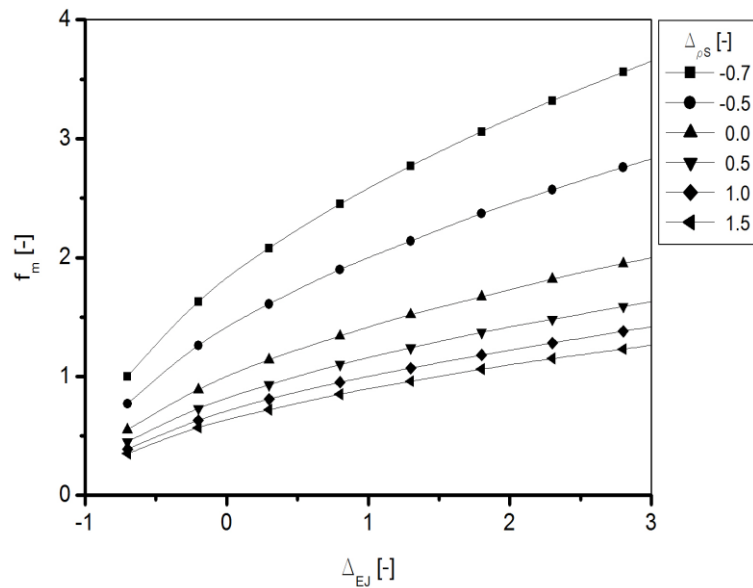

Fig. 3 *The dependency of f_m on Δ_{EJ} for different mass modification $\Delta_{\rho S}$*

Determination the cores geometrical parameters and material properties to achieve required value of the modification function, resp. required natural frequency:

For this purpose, two identical cores situated symmetrically to planes *xy* and *zy* are considered. To define the geometrical parameters value of the modification function f_m is selected (Fig. 4 - dashed line). The required cross section area of the both cores ($S_1 = S_2$) and their position to the neutral axis of the beam structure ($h_1 = h_2$) are determined by using a

corresponding curves (for selected value $\Delta_{EJ(k)}$) to determination of dimensionless modification parameter $\Delta_{\rho S}$. The dimensionless mass modification $\Delta_{\rho S}$ with dimensionless stiffness modification $\Delta_{EJ(k)}$ for the defined parameters δ_E and δ_ρ ($E_1 = E_2;\ \rho_1 = \rho_2$) can be expressed (Fig. 4a) by following schema

$$f_m(\Delta_{EJ}, \Delta_{\rho S}) : \Delta_{EJ(k)} \rightarrow \Delta_{\rho S} , \tag{19}$$

The dimensionless cross section area δ_S can be expressed from following expression

$$\delta_{S,1} = \frac{\Delta_{\rho S}}{n(\delta_{\rho,1} - 1)} , \quad \text{for given purpose} \Rightarrow n = 2. \tag{20}$$

Then, the cross section of the both cores can be determined from (13). The position of the both cores to the neutral axis of the modified beam structure ($h_1 = h_2$) is obtained from the following relation

$$h_1 = \sqrt{\delta_{\xi,1}\xi^2 - \xi_1^2} , \quad \text{where} \quad \delta_{\xi,1} = \frac{\Delta_{EJ(k)}}{n(\delta_{E,1} - 1)\delta_{S,1}} , \quad \text{for given purpose} \Rightarrow n = 2. \tag{21}$$
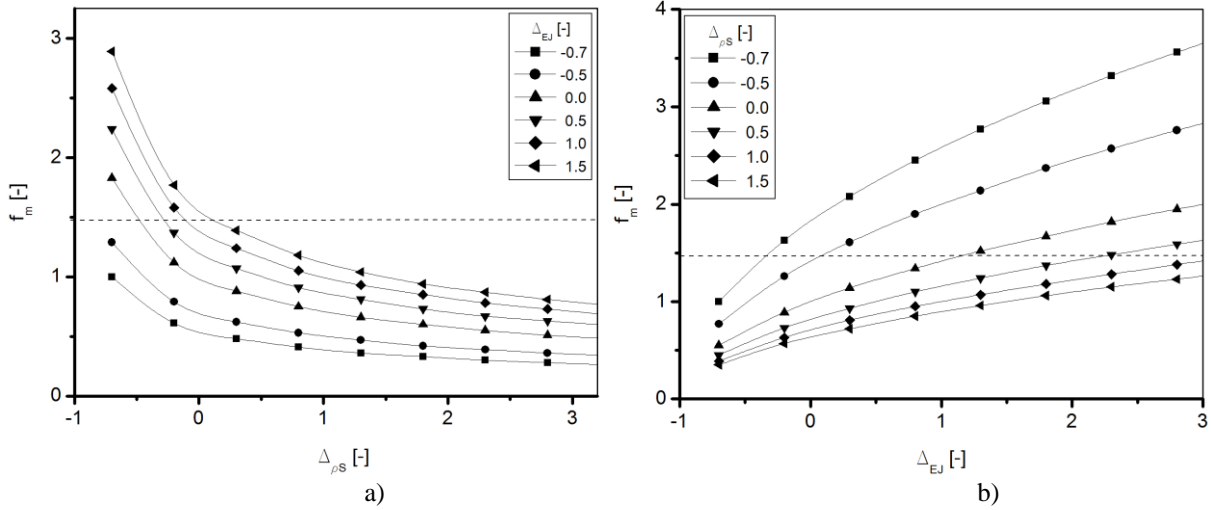

Fig. 4 *Determination of core parameters*

The similar approach can be used to define the material properties of the reinforcing cores ($E_1 = E_2;\ \rho_1 = \rho_2$) for given geometrical parameters.

CONCLUSION

The modal properties analysis of beam structures modified by cores is presented in this paper. The main aim of analyses was to determine the dependency of the beam natural frequencies on material properties and geometrical parameters of reinforcing cores. The natural angular frequency of modified beam structure can be determined by multiplying of natural angular frequency unmodified beam structure and modifying function,

$$\omega_{m,i} = f_m(\Delta_{\rho S}, \Delta_{EJ})\omega_i , \quad \text{for } i = 1, 2, ..., \infty, \tag{23}$$

The modifying function $f_m(\Delta_{\rho S}, \Delta_{EJ})$ considering all the relevant structural changes of beam body and cores parameters is derived. For the defined dimensionless mass and stiffness modification, the function $f_m(\Delta_{\rho S}, \Delta_{EJ})$ has the same value for all natural angular frequencies. This manner of the beam structural modification offers very effective tool to modification of dynamical properties or dynamical tuning for similar beam structures.

ACKNOWLEDGMENT

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