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ACTIVE CONTROL OF OSCILLATION PATTERNS IN NONLINEAR DYNAMICAL SYSTEMS AND THEIR MATHEMATICAL MODELLING

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ABSTRACT

The article deals with the active control of oscillation patterns in nonlinear dynamical systems and its possible use. The purpose of the research is to prove the possibility of oscillations frequency control based on a change of value of singular perturbation parameter placed into a mathematical model of a nonlinear dynamical system at the highest derivative. This parameter is in singular perturbation theory often called small parameter, as $\varepsilon \to 0^+$. *Oscillation frequency change caused by a different value of the parameter is verified by modelling the system in MATLAB.*

KEY WORDS

Nonlinear dynamical system, oscillation control, MATLAB

INTRODUCTION

Occurrence of oscillations, encompassed in a wide range of interesting phenomena, provides a basis for their active control to be an important issue. Various examples can be found in the areas of mechanics and mechanical engineering, electrical engineering and telecommunications, chemistry and chemical engineering, biology, biochemistry and medicine, or economics (see, e.g. (1), (2), (7) and the references therein).

In any case, the concept of control should be composed of five basic elements:

1. The controlled system; in our case described by differential equations.
2. The control goal, generally subdivided into regulation and tracking. Regarding oscillation control, two more objectives need to be taken into consideration, viz. synchronization and modification of asymptotic behaviour of the system. The latter one represents, inter alia, the creation of oscillations with desired properties (e.g. frequency and other parameters of intensity).
3. The set of measurable variables.
4. The set of controlling variables, i.e. variables affecting the system behaviour after being subjected to a change; in our case represented by the singular perturbation parameter mentioned above.
5. The controller enabling used control inputs to achieve the desired goal.

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Our research aims at proving that a singular perturbation parameter $\varepsilon \to 0^+$ plays a crucial role as a modelling tool for the frequency control of a nonlinear oscillations stemming out in nonlinear dynamical systems.

METHODS

Oscillation control can generally be stated in terms of two objectives:

(a) To obtain an asymptotically stable zero solution attracting all initial conditions in a suitably large region (regulator problem (4).

(b) To obtain an asymptotically stable periodic solution with desired properties (such as oscillation at the given amplitudes and frequencies) and which attracts all initial states in a suitably large region (oscillator problem) (2), (3).

In this context, usually two-well potential of an unperturbed system was considered by using analytic methods and numerical simulations (2), (8).

A basic tool used for results of analysis verification is MATLAB and its implemented functions for solving differential equations. Initial value problems can be solved using Newton-Raphson method implemented in ode45 and ode23 used for nonstiff differential equations, Adams method, commonly known as ode113, used for nonstiff differential equations, ode15s (NDFs method), ode23s (Rosenbrock method) and ode23tb (TR-BDF2 method) used for stiff differential equations, Trapezoidal rule represented by ode23t solver used for moderately stiff differential equations and ode15i (BDFs method) used for fully implicit differential equations. Bvp4c and bvp5c are the most effective boundary value problems solvers for differential equations.

The Newton-Raphson method implemented into MATLAB as ode45 is commonly considered to be the most effective means of computing the solution vector of n-dimensional system of nonlinear equations (5), (6). General syntax for all ode solvers is as follows:

[T,Y] = *solver*(odefun,tspan,y0) [1]

Solver expresses a particular solver used in computing the result, in our case ode45. *Odefun* represents the system of differential equations rewritten into basic form *dydx = odefun(x,y)*, where *x* is a scalar, *dydx* and *y* are column vectors. *Tspan* stands for a vector specifying the interval of integration from time 0 to time final. If the vector contains two elements, i.e. it has the form of *[t0 tf]*, the solver returns the solution evaluated at every step of integration. Finally, *y0* represents a vector of initial conditions.

In this paper, we focus our attention on the existence of nonlinear oscillations in the dynamical system describing the singularly perturbed forced oscillator of Duffing's type with a nonlinear and nonperiodic external driving force:

$$\varepsilon^2(a^2(t)y')' + f(y) = m(t), 0 < \varepsilon \ll 1 \quad (8). \qquad [2]$$

Rewriting the equation into a system of autonomous equations, applying the limit $\varepsilon \to 0^+$ for reducing the system into an algebraic-differential reduced system followed by introducing the substitution $\tau = \frac{t}{\varepsilon}$ and obtaining the so-called associated system, provides the methodical base for applying singular perturbation theory, as both systems agree on the level of phase space structure when $\varepsilon \neq 0$, but differ strongly for $\varepsilon = 0$. (For detailed mathematical foundations of these steps, see (7) and (8)).

RESULTS

By way of example, let us consider a second-order differential equation (Duffing's equation) of the form

$$\varepsilon^2 y'' - 3y + y^3 = -t \qquad [3]$$

subject to the initial conditions $y(-4) = 3.1821, y'(-4) = 0$ on the interval $t \in \langle -4; 4 \rangle$ (7).

In order to obtain a solution of the equation [3], it is firstly necessary to rewrite it into a system of two first-order differential equations, as follows:

$$y'' = \frac{1}{\varepsilon^2}(-t + 3y - y^3) \qquad [4]$$

Applying the substitution $Y_1 = y, Y_2 = Y_1' = y', Y_3 = Y_2' = y''$, the equation [4] is rewritten into:

$$Y_1' = \frac{Y_2}{\varepsilon}$$

$$Y_2' = (-t + 3Y_1 - Y_1^3) \qquad [5]$$

Consequently, we are able to create a MATLAB function computing the original equation [3] as follows; the resulting graph is presented in the Figure 1:

```
function equation               %function name

ic1=3.1821;                     %initial conditions
ic2=0;
t0=-4;                          %time interval from t0 to tfinal
tf=4;

[t,y]=ode45(@system1, [t0 tf],[ic1 ic2]); %ode45 solver syntax

plot(t,y(:,1),'k');             %plotting the function
xlabel('t');
ylabel('y');

function dy=system1(t,y)        %the system of first-order ODEs [4]

eps=0.01;
dy=[y(2)/eps; (-t+3*y(1)-y(1)^3)];
end

end
```

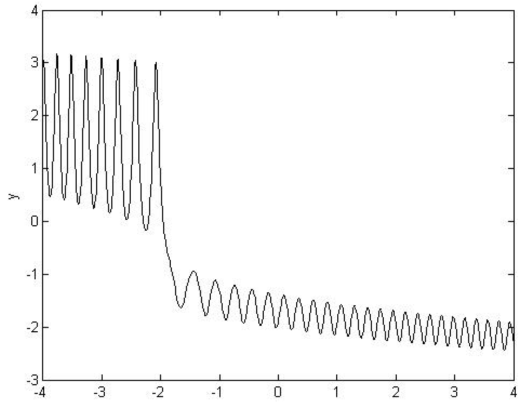
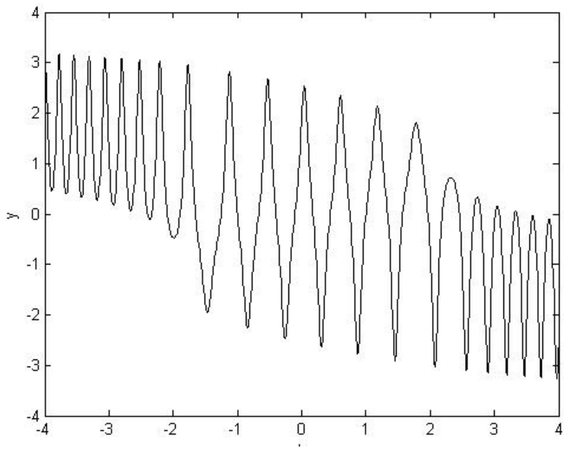
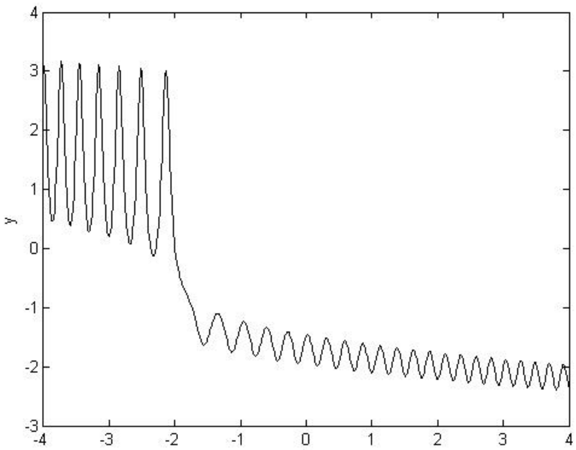


Fig. 1 *Solution of the equation [3] for* $\varepsilon^2 = 0.01$

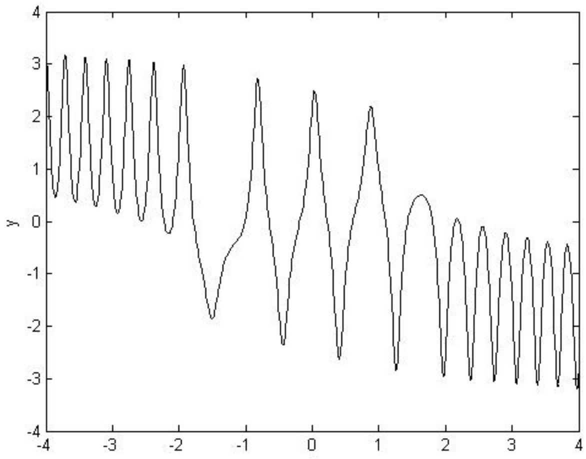
Varying the singular perturbation parameter ε, the graph of the equation [3] differs significantly. It is necessary to point out, that these oscillations are very sensitive on the value of singular perturbation parameter ε, as it comes out from the following figures. To compare, the difference of the parameter ε in Figure 1 and Figure 2(a) is on the third decimal place, hence the oscillations are extremely different.



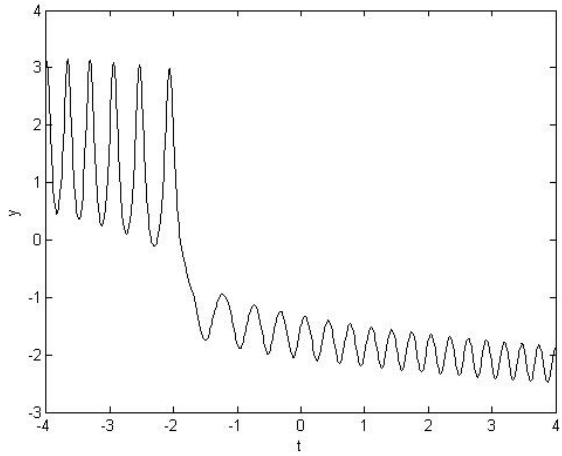
(a)



(b)



(c)



(d)

Fig. 2 *Solution of the equation [2] for* $\varepsilon^2 = 0.009$ *(a),* $\varepsilon^2 = 0.013$ *(b),* $\varepsilon^2 = 0.015$ *(c),* $\varepsilon^2 = 0.02$ *(d)*

DISCUSSION

The results imply the hypothesis that different frequency oscillations can be achieved as a result of the change of the singular perturbation parameter ε at the second derivative. When comparing the Figure 1 with Figure 2, it is obvious, that the oscillations differ significantly, although there is only a minor change in the parameter ε .

CONCLUSION

We demonstrated the possibility of oscillations control based on a small fitting parameter placed into a mathematical model of a nonlinear dynamical system. The results were modelled using MATLAB software. However, deep understanding of each individual system is required in order to obtain oscillations with specific properties requires. It also depends, inter alia, on particular system-behaviour requirements and initial conditions and represents our future research questions needed to be answered.

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