RESEARCH PAPERS
FACULTY OF MATERIALS SCIENCE AND TECHNOLOGY IN TRNAVA
SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA

2016 Volume 24, Number 38

PART AS AN OBJECT OF ASSEMBLY

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Abstract

We deal with static prearranged couples in practice quite often. Constructers are using them a lot to improve rigidity of product. Some examples are mentioned in this article. The paper also discusses the models of kinematic couples in plane and models of kinematic couples in space. Spatial movable and non-movable couples are commented on as well. The article particularly focuses on application of statics in the assembly in spatial couples (movable, non-movable) for intentional allowance fix which leads to a non-problem assembly. With fix like this, it is possible to manufacture selected parts in series while assuring non-problem assembly.

Key words

assembly, spatial movable, unmovable couples, Grübler's equations

INTRODUCTION

Aim of this paper is application of statics in the assembly, especially in spatial couples for intentional allowance fix which leads to non-problem assembly. Dealing with static prearranged couples is common in real life.

STATICS IN ASSEMBLY

Each designer knows that assembled structures can be analysed either as planar (if the thickness of the members are negligible, as if they were made of a metal sheet) or spatial. Textbooks of statics offer the so called Grübler's equations for both groups to calculate the number of degrees of freedom (Σk) of a couple or a multi-structure (1, 2).

Typical example where constructers use static prearranged couples can be ball bearing which should have theoretically three balls in maximum capacity. However, each bearing contains more balls because of the mentioned rigidity improvement.

Grübler's equations can be used for example for creation of multi-DOF closed kinematic chains, where one of the links is stopped and total DOF of such system becomes one (10).

Another example can be solution to the structural synthesis problem in the design of manipulators (11).

Couples in the textbooks are drawn in their structural design. More accurate, however, would be just drawing a point model instead of the real one (Figs. 1 and 2).

Statics in fact address the balance of forces in the models where reactions act in single points (3).

To detect static properties of the examined structure, the latter will be divided into the following groups:

1. **Plane:** These are the structures made of the members so thin that their thickness can be neglected (Fig. 1), or their thickness is real, but the members are symmetrically distributed on the functional plane.

2. **Space:** They are characterised by the fact that all moving members perform general spatial movements (Fig. 2).

Using statics, the reactions can be investigated between the rotating circle (pin) and prism into which it is inserted (two reactions). The reactions can also be examined between the ball and the inside of a triangular pyramid (ball pin) as shown in Figure 2 line 3.

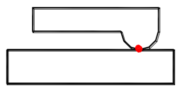
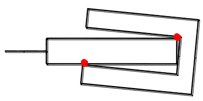
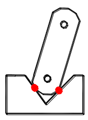
Model	**Σ k**	**Name**
	**1**	**One point couple**
	**2**	**Sliding couple**
	**2**	**Rotational couple**

Fig. 1 *Models of kinematic couples in a plane, where: Σk – number of degrees of freedom*

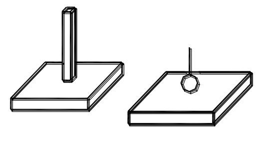
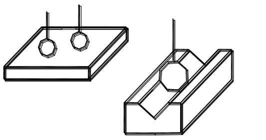
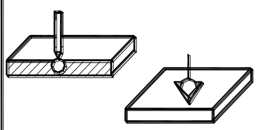
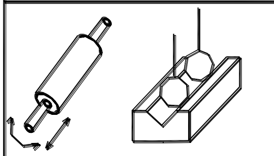
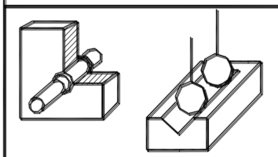
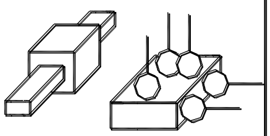
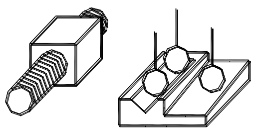
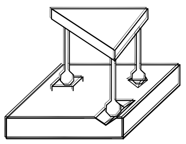
Model	k_p	Name
	**1**	**Single-point couple (a point on the plane)**
	**2**	**Bipod (sphere in the groove on the space curve)**
	**3**	**Tripod (ball joint)**
	**4**	**Rotary–sliding couple**
	**5**	**Rotary couple**
	**5**	**Sliding couple**
	**5**	**Screw couple**
	**6**	**Unmoveable definite static couple**

Fig. 2 *Models of kinematic couples in space, where:* Σk *- number of degrees of freedom*

The necessity to display couples using their point models should be underlined; otherwise, the well-known Grübler's equations could be misleading.

Example 1: In order to calculate i_r for the structure in Figure 3.

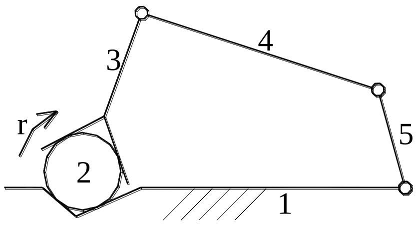


Fig. 3 *Plane mechanism*

According to Grübler, the equation gets the following shape (3):

$$i_r = 3_n - 3 - 2.r - 2.p - 1.o \quad [-] \qquad [1]$$
$$i_r = 3_n - 3 - r.2, \text{ i.e. } i_r = 3.5 - 3 - 5.2 = 2 \ [-] \,,$$

where: i_r – number of degrees of freedom in the plane [–],
n – number of members [–],
r – number of rotational couples [–],
p – number of sliding couples [–],
o – number of general (one-point) couples [–].

It is obvious that it is a planar mechanism where $i_r = 1$. Grübler does not consider the structures which contain rotating members (member 2). In such case, it is necessary to deduce the number of members with their own rotation (Σr) from the calculated i_r. The example illustrates one own rotation representing one degree of freedom, which is not, however, manifested in the behaviour of the structure as a whole.

In the current monograph, the author used the equations derived by Valentovič (4).

A prerequisite for the calculation of i_r is drafting a point model with reactions in single points.

The equation of Valentovič for the plane takes the following shape:

$$i_r = 3_n - 3 - \Sigma_k - \Sigma_r \quad [-] \,, \qquad [2]$$

where: n – number of members [–],
Σ_k – number of contact points in a given model [–],
Σ_r – number of members rotating in themselves [–].

The procedure is similar in the case of space. Figure 2 shows the models of selected spatial couples with marked discrete points of contact.

The equation for space takes the form (4):

$$i_p = 6n - 6 - \Sigma_k - \Sigma_r \quad [-] \,. \qquad [3]$$

Example 2: According to Grübler, the crank mechanism has one degree of freedom, because it is considered a planar mechanism with the members of zero thickness. Members of a real crank mechanism move in several parallel planes. Such structures are spatial (Fig. 4); they must be therefore investigated by the Valentovič equations for space:

$$i_p = 6.4 - 6 - (3.5.1.5) - 0 = -2 \ (!) \ [-] \,,$$

i_p = +1 was anticipated by the author. The structure in space is, however, redetermined three times (+1) - (-2) = 3. Our experience confirms that when closing such structure, e.g. for bonding a connecting rod with a crosshead, it is necessary to extend the gap between the pin and the hole.

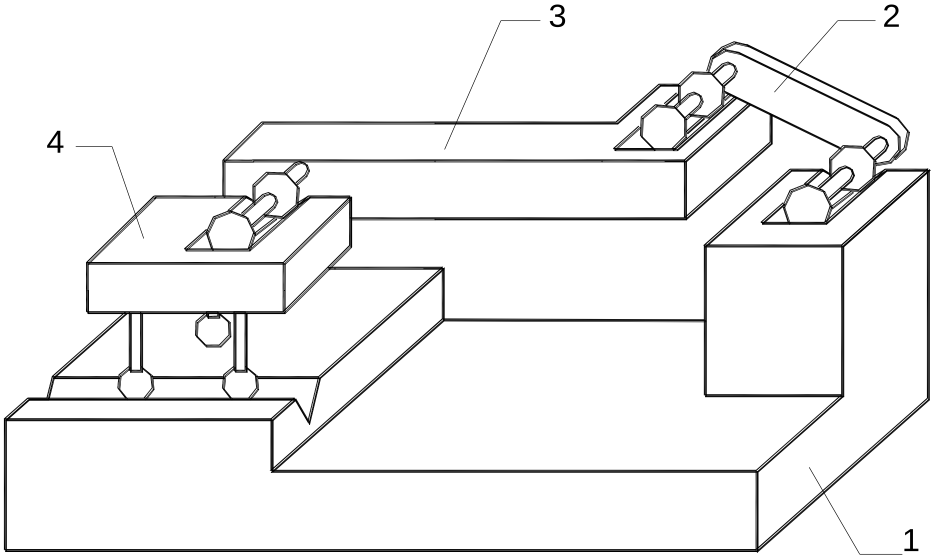


Fig. 4 *The crank mechanism is considered a system with one degree of freedom (i_p =1). A detailed analysis shows i_p = -2. The structure is considered spatial*

Redundant degrees of freedom can be eliminated for example by replacing three rotating couples (k = 5) by rotary - sliding couples (k = 4).

The equation to calculate the number of degrees of freedom of each structure is [3], based on the spatial model, since planar structures do not exist in practice.

The number of degrees of freedom depends on the purpose of the structure. The differential should have i_p = 2, mechanism should have i_p = 1 and unmovable structure i_p = 0.

The structures assembled in a trouble-free way are:

$$ip \geq 0.$$

The structures assembled in a troublesome way are:

$$ip < 0.$$

The difference between the anticipated i_p and calculated i_p is called the degree of redetermined structure. The higher the degree of redetermined structure, the more complex the calculation of the gaps necessary to ensure trouble-free assembly is.

The structure is statically redetermined if the number of degrees of freedom (i_p) calculated from the equation [3] is smaller than anticipated (i_p = 0 is anticipated for an unmovable structure, i_p = 1 for mechanisms and i_p = 2 for differentials).

If the structure is redetermined, its expected function will only be achieved by a suitable toleration treatment which is dealt by in the theory of strings (5, 6).

Frequently, neither the expected result of the equation [3] can guarantee a smooth assembly.

Statics, as presented in textbooks, are not suitable for the purposes of this paper due to the principal shortcomings identified (3).

This paper is thus also a contribution to the development of statics.

Grübler’s equations are considered crucial for statics, despite their shortcomings. They should be, however, replaced by Valentovič’s equations applied in this paper (7, 4).

SPATIAL MOVABLE AND UNMOVABLE COUPLES

Basic structures are couples or bonds of two bodies. Figure 5 gives a basic overview of the couples used in engineering: a ball joint, a rotary sliding couple, a rotary couple, a sliding couple A, a sliding couple B and a screw couple.

Professor Whitney who also deals with this issue uses the real shapes of bodies to show the couples, which are not compatible with the theory of statics (8).

The author agrees with Professor Valentovič who promotes the use of spherical models that are compatible with statics, unlike photo images and drawings showing engineering couples and structures (7, 4).

Advantages of such an approach are its transparency and closeness to the language of "statics engineers".

Figure 5 shows the schemes of couples in a shape of spherical models, and their symbols used in drawing the whole structure in order to avoid making it laboriously of spherical models.

The principle of a spherical model is that bodies, in accordance with statics, are perceived as perfect solids touching not in planes but in single points.

Figure 5 shows that a ball joint in a ball bed, drawn as a ball model is a three-point model (touching the other body at three points), the rotary sliding couple is a four-point model, the rotary couple is a rotary sliding model prevented in the movement (feed), i.e. a five-point model, and the sliding couple A and B as well as the screw couple are five-point models. The spherical models are accompanied by symbols.

Such couples are correct and can be easily assembled when made according to the spherical models. However, the couples may be also incorrect, and special attention should be paid to the couples of gears which are generally considered as rolling. This is not true however, since if one tooth of a wheel engages the notched wheel gap of the other wheel according to the scheme (Fig. 6d) at a given axial distance, the notched tooth gap touches only one point. The tooth will touch two points only if the wheel is released and pushed into the other wheel (Fig. 6e).

Special attention should be paid to this phenomenon because of a generally accepted misconception that the gears always form a single-point (i.e. rolling) couple.

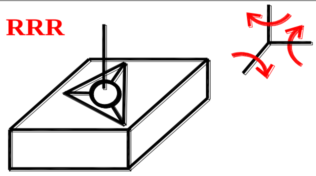
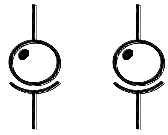
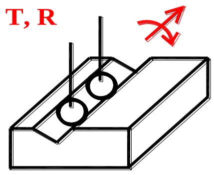
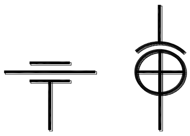
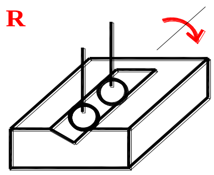
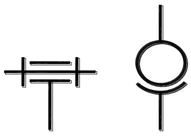
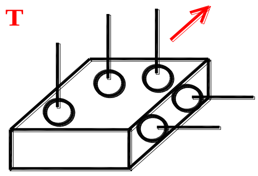
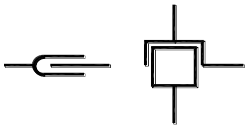
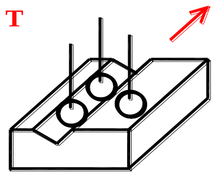
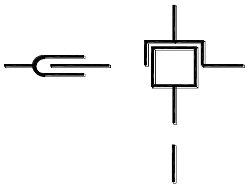
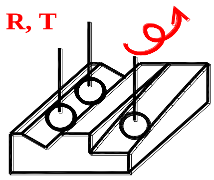
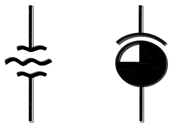
	3D	2D	Σ k	Name
a			**3**	**ball point**
b			**4**	**rotary sliding couple**
c			**5**	**rotary couple**
d			**5**	**sliding couple A**
e			**5**	**sliding couple B**
f			**5**	**screw couple**

Fig. 5 *Basic space movable non-singular couples.*
Proposal for standardisation of signs.

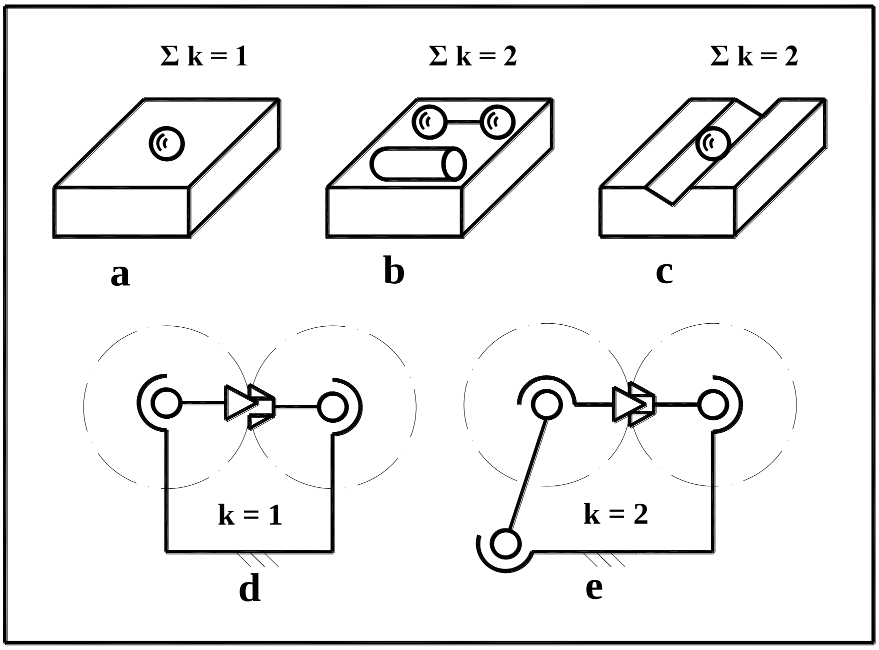


Fig. 6 *Movable rolling couples (a, b, c) and tooth couples (d, e)*

In the case of rolling couples, it is important to distinguish whether a ball rolls on a plane (Fig. 6a), a prism rolls on a plane (Fig. 6b) or balls roll in a V-groove (Fig. 6c). It is quite obvious that in each case there is a different number of contact points (9).

CONCLUSION

Statics in the form as we can see in the textbooks is not usable for the purposes of assembly. Reasons are principal shortcomings which were mentioned above (3).

This article also contributes to the progress and development in statics.

Grübler's equations are the key equations of statics. Yet, they have some limitations and they should be, therefore, replaced by Valentovič's equations since the latter suit better for the purposes discussed in this paper (7, 4).

Usage of ball models which are compatible with statics, in contrast to photographic pictures and drawings for displaying mechanical couples and structures, is the right idea (8).

This approach is better because of compatibility with the statics theory.

Principle of a ball model is based on the fact that, in conformity with statics, objects are perceived as perfectly rigid. That is the reason why objects are joined in isolated points and not surfaces. These types of couples are correct and they can be assembled with no problems when they are made according to the ball models.

Acknowledgements

The article was written within the VEGA 1/0477/14 project of "Research of influence of selected characteristics of machining process on achieved quality of machined surface and problem free assembly using high technologies".

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