DYNAMIC STABILITY OF GENERALIZED BECK-REUT`S BEAM

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Abstract

The classical non-conservative Beck`s beam, loaded by follower compressive force, is generalized by allowing an arbitrary angle of action of the follower force as well as allowing for excentric positioning of the applied force. For the corresponding boundary eigenvalue problem, the frequency equation is derived. Results of parametric studies are presented with an emphasis laid on the lowest eigenfrequencies. The characteristic shape of the computed curves indicates whether stability loss by divergence or by flutter occurs. A map of stability is presented in terms of parameters describing the excentricity and the angle under which the follower force acts on the beam.

Key words

dynamic stability, generalized Beck-Reut`s column, divergence, flutter

INTRODUCTION

The Beck`s beam is a classic example of a relatively simple non-conservative system. Transverse vibrations of slender beams is a well-understood object, conservative axial end loads can lead to buckling, while follower forces can introduce flutter as a typical phenomenon in non-conservative systems. The classical introduction to engineering theory of non-conservative systems is the well-known Bolotin`s monography (1), and mathematical theory detailed in Leipholz`s monography (2). An extensive critical overview based on hundreds of papers has been presented by Elishakoff (3). Koiter (4) questioned the usefulness of the study of follower forces due to a lack of their industrial applications, her paper provoked a series of responses (5-6, 3). To conduct a study of follower forces in a laboratory is a quite challenging task (7), one of the few implementations of follower forces uses reactive forces at the tip of the beam or fluid stream ejected from a nozzle attached to the free end of the beam. However, follower forces can be easily implemented in MEMS and, as stated (8), provide performance enhancement of certain MEMS devices. Frequency tuning of a microscale cantilever beam has applications in

such modern applications, including atomic force microscopes and high density data storage devices (9).

The cantilever beam loaded by a compressive force **P** of constant magnitude gives rise to different types of behavior depending on the way the load interacts with the beam structure. The most common case is given by the "dead" load, see Fig. 1a, when the direction of the compressive force maintains its orientation regardless of the beam deflections. Gravitational forces are a typical example, the corresponding boundary problem is self adjoint and as far as the elastic stability is concerned, the Euler`s classical approach gives satisfactory results. In other words, the static criterion of stability can be used. Beck`s beam is a cantilever loaded by a compressive force oriented in a direction tangential to the deflection curve, Fig. 1b. This type of load is labeled as a follower force. Beck`s beam is a non-conservative system. To assess its stability the dynamic criterion of stability is required in which inertia effects are included. The loss of stability in the case of a conservative dead load corresponds to divergence, while under
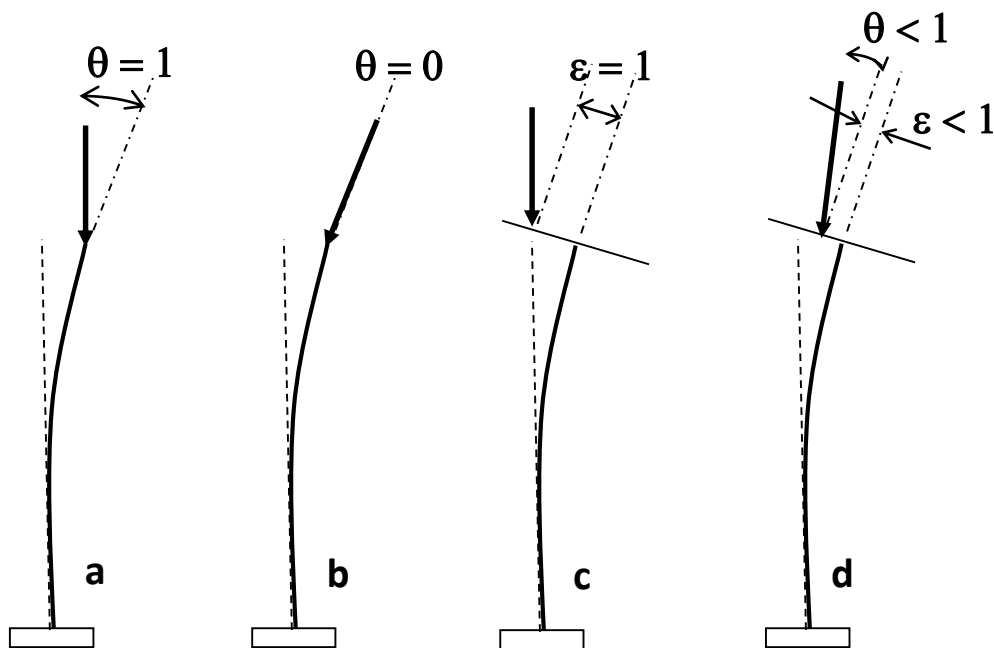

Fig. 1 *Different types of behavior of the compressive force: a – dead load, b – Beck`s column, c – Reut`s column, d – generalized Beck-Reut`s column*

the follower force, the mechanism of stability loss may result in flutter – i.e., vibration with increasing amplitude. An attractive feature of the Beck`s column is the fact, that the theoretical critical compressive force is more than eight times higher than the critical force in the conservative case. Reut`s column arises when the compressive force maintains both the direction and the line of action, Fig. 1c. A platform at the beam tip is required to keep its interaction with the deflecting beam. Here due to the excentricity ε additional moment acts at the beam end. This case is also non-conservative, moreover, in mathematical terms the Reut`s column is adjoint to the Beck`s column and as such has the same critical loads as the Beck`s column. Generalization of the above mentioned three beams can be obtained if we allow a sub tangential compressive force together with partial excentricity as illustrated in Fig. 1d. The aim of this paper is to study the influence of the angular declination θ and of the excentricity ε on the lowest eigencurves and to draw conclusions on possible transition from divergence mechanism to flutter mechanism of stability loss.

GOVERNING EQUATIONS

We consider a straight slender elastic prismatic clamped beam loaded at its end by a compressive force **P** of constant magnitude *P*. The directional features of the behavior of the load vector with respect to the deflecting beam are specified via the boundary conditions. The governing equation for a constant cross-section and for harmonic vibration in a single plain is:

$$y^{(4)}(x) + \alpha y^{(2)}(x) = \lambda^2 y(x), \qquad [1]$$

where $y(x)$ is the non-dimensional flexural deflection, $\alpha = Pl^2/EJ$ is the constant non-dimensional magnitude of the compressive force and $\lambda^2 = \omega^2 m/EJ$ is the non-dimensional frequency with ω standing for circular frequency. The boundary conditions corresponding to the generalized case according to Fig. 1d are:

$$y(0) = 0, \quad y^{(1)}(0) = 0, \quad y^{(3)}(1) + \theta\alpha y^{(1)}(1) = 0, \quad y^{(2)}(1) + \varepsilon\alpha y(1) = 0, \qquad [2]$$

in which the parameter θ describes the directional behavior of the loading vector and the parameter ε is the measure of the excentricity of the point of action of the loading vector with respect to the beam axis. The combination $\theta = 1, \varepsilon = 0$ corresponds to the classical dead load in Fig. 1a, the Beck`s column we have for $\theta = 0, \varepsilon = 0$ and the Reut`s column we obtained for values $\theta = 0, \varepsilon = 1$. The variation of θ in the interval $0 < \theta < 1$ describes the transition from follower force $(\theta = 0)$ to partial (sub tangential) follower force $(0 < \theta < 1)$ and subsequently the conservative dead load $(\theta = 1)$. Similar variation of the parameter ε indicates transition from centric load $(\varepsilon = 0)$ to partially centric load $0 < \varepsilon < 1$ and fully excentric load $(\varepsilon = 1)$.

The frequency equation corresponding to the boundary eigenvalue problem (1), (2) is obtained after rather lengthy calculations as:

$$
\begin{aligned}
&\lambda\left\{\alpha^2 + 2\lambda^2 + \alpha\lambda Sin(k_1)Sinh(k_2) + 2\lambda^2 Cos(k_1)Cosh(k_2) + \right. \\
&\left. + \alpha\theta\left(\alpha Cos(k_1)Cosh(k_2) - 2\lambda Sin(k_1)Sinh(k_2) - \alpha\right)\right\} + \\
&\varepsilon\alpha\left[\alpha\lambda Cos(k_1)Cosh(k_2) + \left(\alpha^2 + 2\lambda^2\right)Sin(k_1)Sinh(k_2) - \alpha\lambda\right] + \\
&\varepsilon\theta\alpha^2\left[2\lambda - 2\lambda Cos(k_1)Cosh(k_2) - \alpha Sin(k_1)Sinh(k_2)\right] = 0,
\end{aligned}
\qquad [3]
$$

where

$$k_1 = \sqrt{\sqrt{\left(\frac{\alpha}{2}\right)^2 + \lambda^2} + \frac{\alpha}{2}}, \quad k_1 = \sqrt{\sqrt{\left(\frac{\alpha}{2}\right)^2 + \lambda^2} - \frac{\alpha}{2}}. \qquad [4]$$

NUMERICAL RESULTS AND THEIR DISCUSSION

To understand the vibration and stability of the generalized Beck-Reut`s beam, a numerical solution of the frequency equation [3] was performed for a selected sets of parameters θ and ε. Here we present the results for various combinations of parameters θ and ε as series of curves $\alpha = f(\lambda)$ only for the first two eigenfrequencies.

There are basically only two types of eigencurves. In the first type there are two completely separated non-intersecting curves for the first and second eigenfrequency. The loss of stability is due to the mechanism of divergence and here the critical load is traced out by the point of intersection of the lower eigencurve with the vertical axis, where $\lambda = 0$. This is the highest load applicable to avoid loss of stability. Above this load the second eigencurve has only theoretical meaning unless special measures are taken to avoid the divergence.

The second type is characterized by the coalescence of the two lowest eigencurves. At the flutter point the tangent to the eigencurves is horizontal, this is the geometrical interpretation of the flutter point. Immediately above the flutter point there are no more real eigenfrequencies,
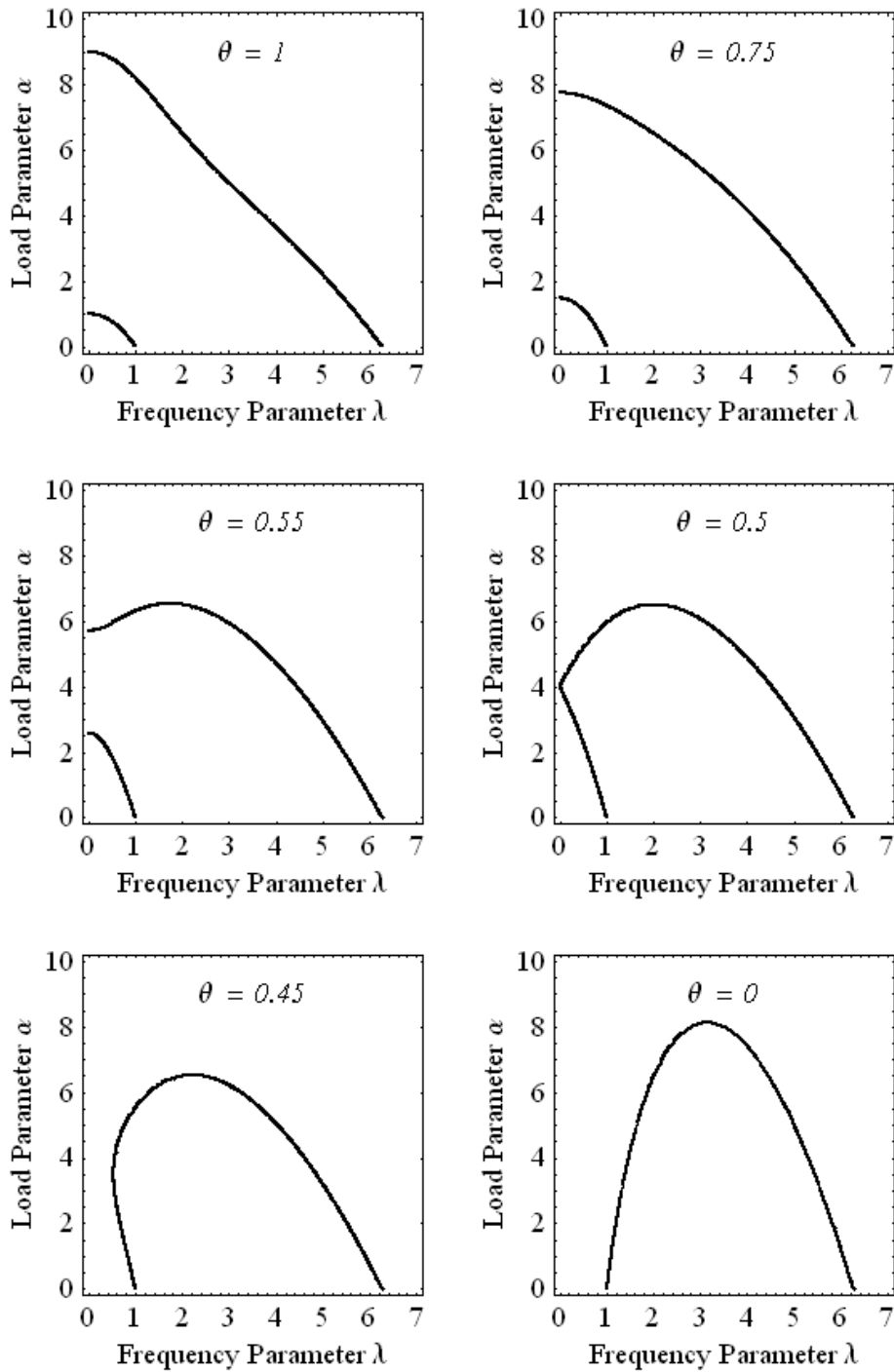

Fig. 2 *Lowest eigencurves for partially follower force with no excentricity*

the lowest two eigenfrequencies are complex and conjugated and the loss of stability is manifested as vibration with increasing amplitude. The flutter point can be computed numerically from simultaneous equations

$$F(\alpha,\lambda,\theta,\varepsilon)=0, \quad G(\alpha,\lambda,\theta,\varepsilon)=\frac{\partial F(\alpha,\lambda,\theta,\varepsilon)}{\partial\lambda}=0, \quad\quad [5]$$

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where θ, ε are fixed and $F(\alpha, \lambda, \theta, \varepsilon)$ is the left hand side of the frequency equation [3]. The explicit form of the function $G(\alpha, \lambda, \theta, \varepsilon)$ is not given here due to its extent, explained mainly by the formulas [4] giving the arguments of the goniometric and hyperbolic functions. Fig. 2 presents the transition of the pairs of the lowest eigencurves from the case of the dead load $(\theta = 1)$ to the Beck`s column $(\theta = 0)$ for the partially follower force applied without excentricity $(\varepsilon = 0)$.
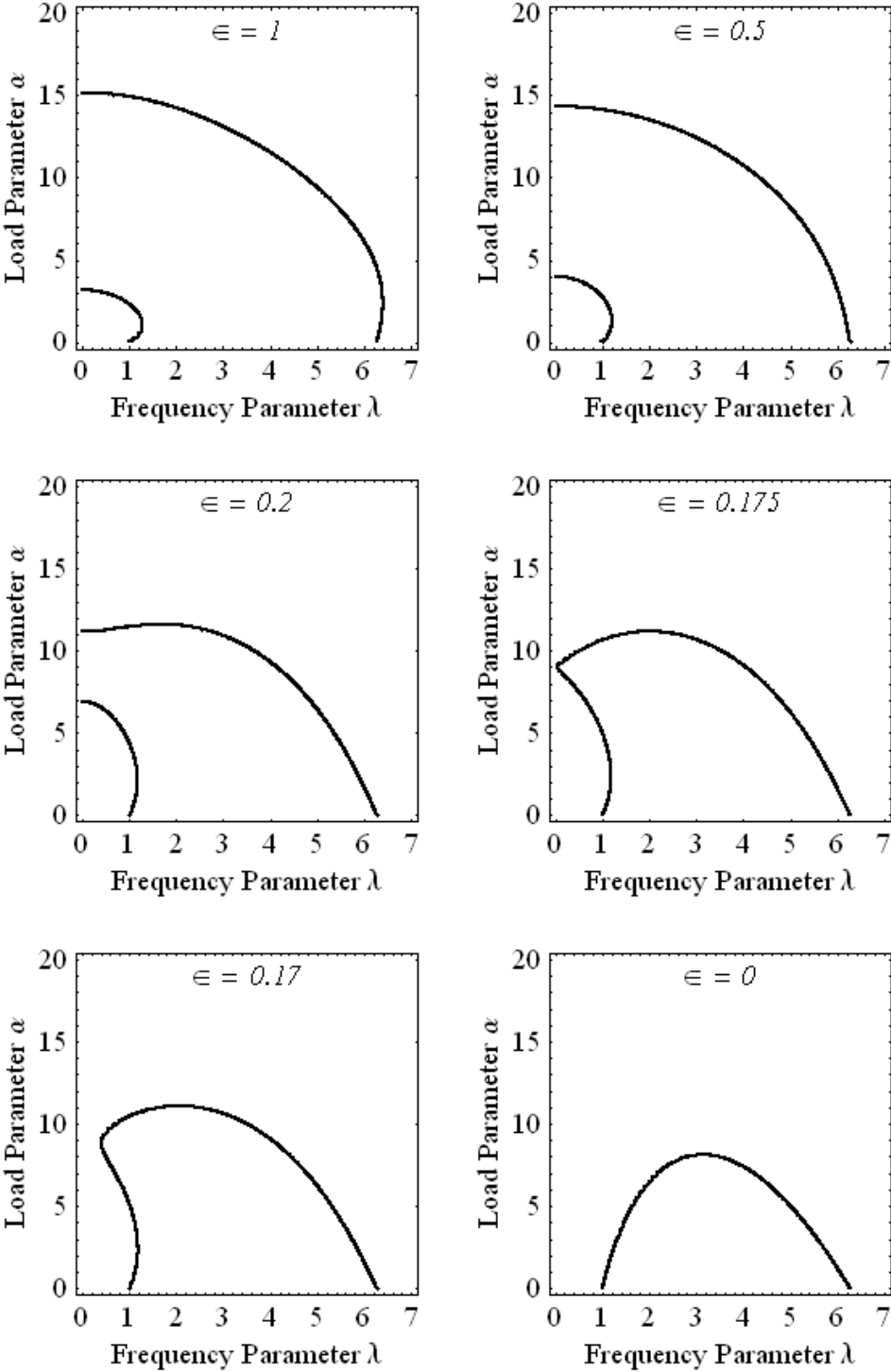

Fig. 3 *Lowest eigencurves for follower force with excentricity*

With decreasing declination θ the critical divergence load increases until the point $\theta=0.5$. Here the critical divergence load suddenly jumps from the value approximately of four to the value of critical flutter load which is slightly above six. With a further decrease of the parameter θ the critical flutter load is increasing until the value of *8.12* corresponding to Beck`s column is achieved. Similar tendencies are valid for the case of follower compressive force *($\theta = 0$),* which is applied excentrically, see Fig. 3. Here the transition between the divergence and flutter occurs at the point $\varepsilon = 0.175$. Of course, with increasing excentricity the tendency to the loss of stability by divergence is stronger.
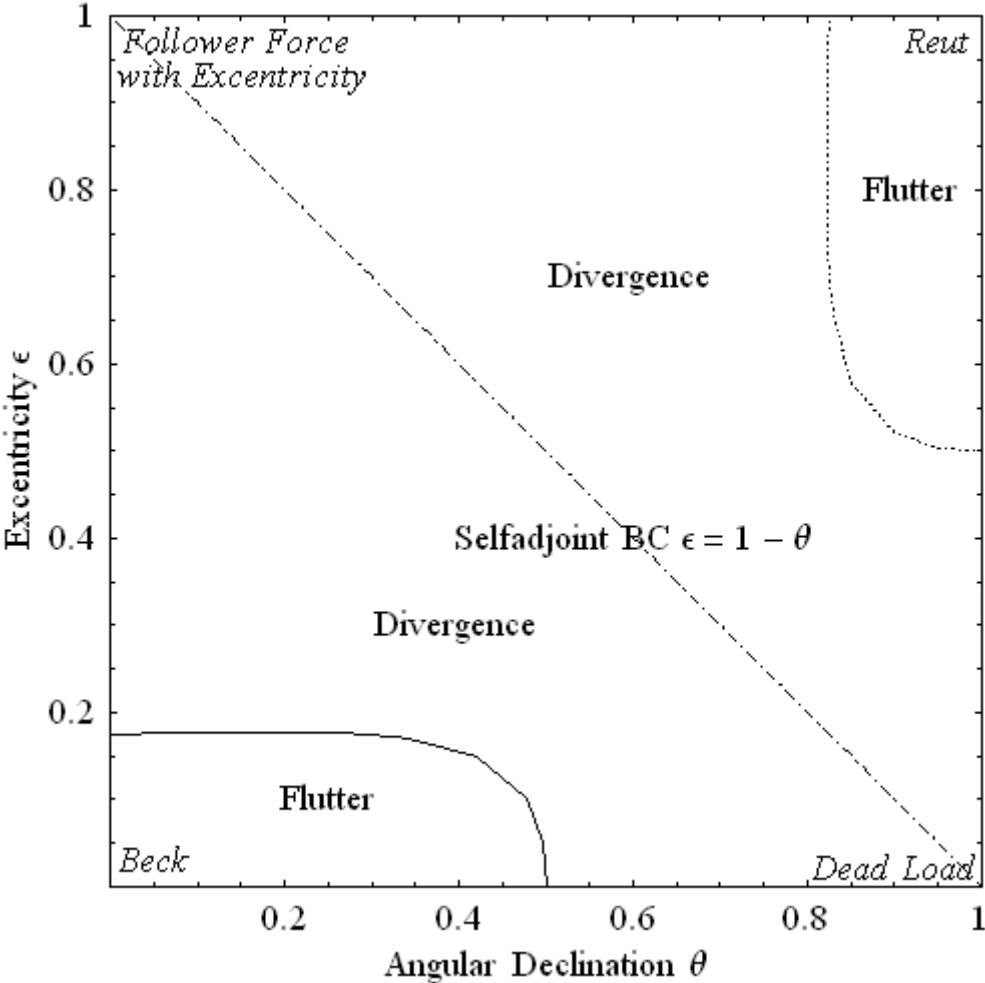

Fig. 4 Stability map for generalized Beck-Reut`s beam

General insight into the qualitative nature of the mechanism of the loss of stability can be obtained from Fig. 4, where the lines of transition from flutter to divergence are indicated. It is interesting, that the results are symmetric with respect to the line $\varepsilon=1-\theta$. One of demonstrations of this symmetry is the fact, that results for the Reut`s beam coincide with those for the Beck`s beam. The equation $\varepsilon=1-\theta$ is the condition for self adjointness of the corresponding boundary eigenvalue problem [1], [2], as derived in (9).

CONCLUSION

The frequency equation and flutter condition has been derived for the generalized Beck-Reut`s beam. The presented parametric studies allow understanding of how the angular declination of the partial follower force, and the excentricity, influence the vibration and the stability of the compressed beam. The stability map shows parameter intervals when flutter or divergence occurs.

Acknowledgements

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