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NOTE ON THE SENSITIVITY OF SIMULATED SOLUTIONS OF THE NONLINEAR SINGULARLY PERTURBED DYNAMICAL SYSTEM ON THE USED DIFFERENT REWRITING INTO A SYSTEM OF FIRST ORDER DIFFERENTIAL EQUATIONS

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Abstract

In this paper we analyze the sensitivity of solutions to a nonlinear singularly perturbed dynamical system based on different rewriting into a System of the First Order Differential Equations to a numerical scheme. Numerical simulations of the solutions use numerical methods implemented in MATLAB.

Key words

singular perturbation, numerical simulation

INTRODUCTION

This paper considers a nonlinear singularly perturbed dynamical system which can be described by the second-order ordinary differential equations with a small parameter ε at the second derivative. The equations are called singularly perturbed differential equations and the theorem of continuous dependence of solutions on parameters is not applicable in this case. These nonlinear systems can be solved by analytical or numerical methods.

In this paper, by using the numerical schemes implemented in the MATLAB environment, we will simulate the nonlinear oscillations in the dynamical system describing the singularly perturbed undamped oscillator with a continuous nonlinear restoring force and without the excitation of oscillatory inputs:

$$\begin{aligned} &\varepsilon^2 y'' + f(t, y) = 0 \\ &y(-\delta) = y_0, y'(-\delta) = y_1 \end{aligned}, \qquad [1]$$

where ε is a small positive parameter and f is a continuous function

$$f(t,y)=\begin{cases} y^{4n+1} & \text{for}\, t\in[-\delta,0] \\ y\prod_{i=1}^{2n}\left(y^2-\mu^2 i^2 h^2(t)\right) & \text{for}\, t\in[0,\infty). \end{cases} \qquad [2]$$

$[y_0, y_1]$ is an initial state, $y_\varepsilon(.,y_0,y_1)$ is a direct output, h is a positive continuous function on $[0,\infty]$, $n\in N$, $\delta>0$, and ε, $0<\varepsilon<<1$ is a singular perturbation parameter. It is instructive for the future to keep in mind the symmetric pitchfork-shaped manifold $f(t,y)=0$. The parameter $\mu>0$ is a constant determining the distance between pitchfork arms (1).

This second order ordinary differential equation in general can be described as a system of first order equations of the form:

$$\begin{aligned} y' &= \frac{z}{\varepsilon^p} && \textit{where}\, \mathrm{p}+\mathrm{q}=2 \\ z' &= -\frac{f(t,y)}{\varepsilon^q} && \textit{for}\, \mathrm{p},\mathrm{q}\in(0,\infty) \end{aligned} . \qquad [3]$$

NUMERICAL SIMULATION

There are several major types of practical numerical methods for solving initial value problems for ODEs in the MATLAB environment. We used solvers based on Runge-Kutta methods:

- *ode45*: This numerical solver is based on an explicit Runge-Kutta [4, 5] formula. That means the numerical solver *ode45* combines a fourth order Runge-Kutta method with a fifth order error control. *ode45* is a versatile ODE solver and is the first solver you should try for most problems.
- *ode23*: This numerical solver is based on an explicit Runge-Kutta [2, 3] formula. The solver is used for problems with crude error tolerances or for solving moderately stiff problems (4).

Numerical analysis of this type of equation show the high sensitivity on the initial conditions, the perturbation parameter and the used numerical method (2, 3).

We simulate solutions of the nonlinear dynamical system [1], when a continuous function [2] is in the form:

$$f(t,y)=\begin{cases} y^5 & t\in\langle-\delta,0\rangle \\ y\prod_{i=1}^{2}\left(y^2-0.5^2 i^2 0.5^2 t^6\right) & t\in\langle 0,\infty\rangle \end{cases},$$

where $n=1$, $h(t)=0.5t^3$, $\mu=0.5$.

When $0\le p<<2$ in the system of first order equations [3], simulation of the numerical solutions are the same, as demonstrated by simulation in the MATLAB on Figs. 1-3 (solver ode45) and Figs. 8-9 (solver ode23). The first equation of [3] shows the first derivation. The first derivation is the slope of the tangent line to the function at point x. When p is near zero, the slope is not so large and the simulation is more realistic.

When $0 < p < 2$ we can see high sensitivity of solutions on p and the used numerical method, is demonstrated by simulation in MATLAB shown in Figs. 4-8 (solver ode45) and Figs. 11-12 (solver ode23).

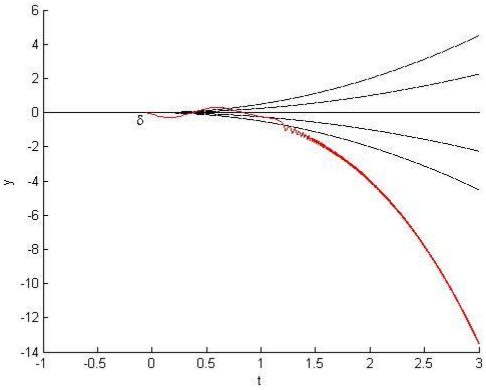

Fig. 1 *ode45: Numerical solution of (1), (2),* $y(-0.05)=0, y'(-0.05)=-1.81, n=1, \mu=1,$ $\varepsilon=0.01, h(t)=0.25t^3, p=0, q=2.$

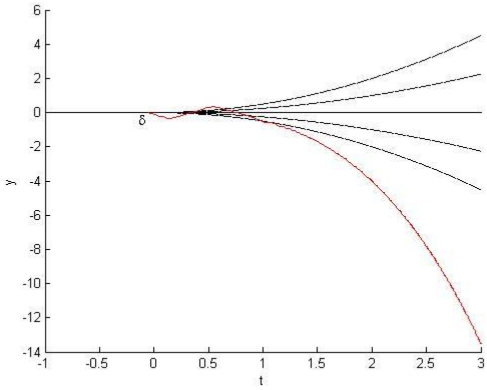

Fig. 2 *ode45: Numerical solution of (1), (2),* $y(-0.05)=0, y'(-0.05)=-1.81, n=1, \mu=1,$ $\varepsilon=0.01, h(t)=0.25t^3, p=\frac{1}{40}, q=\frac{79}{40}.$

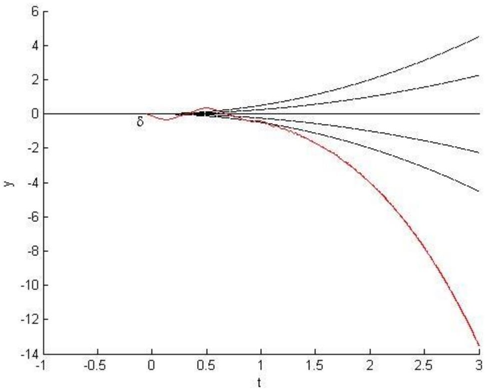

Fig. 3 *ode45: Numerical solution of (1), (2),* $y(-0.05)=0, y'(-0.05)=-1.81, n=1, \mu=1,$ $\varepsilon=0.01, h(t)=0.25t^3, p=\frac{1}{20}, q=\frac{39}{20}.$

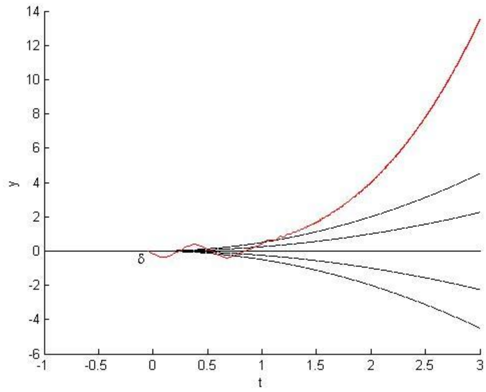

Fig. 4 *ode45: Numerical solution of (1), (2),* $y(-0.05)=0, y'(-0.05)=-1.81, n=1, \mu=1,$ $\varepsilon=0.01, h(t)=0.25t^3, p=\frac{1}{8}, q=\frac{15}{8}.$

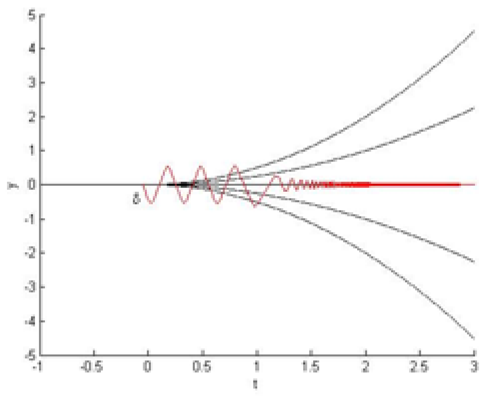

Fig. 5 *ode45: Numerical solution of (1), (2),*

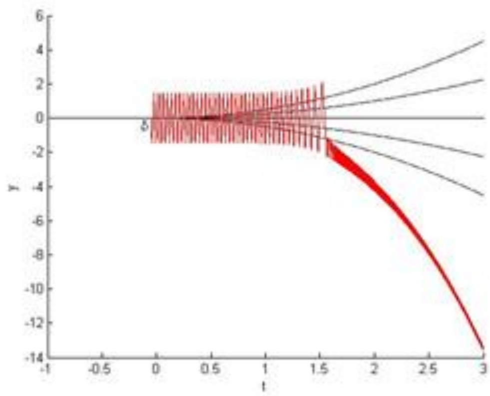

Fig. 6 *ode45: Numerical solution of (1), (2),*

$y(-0.05)=0, y'(-0.05)=-1.81, n=1, \mu=1,$
$\varepsilon=0.01, h(t)=0.25t^3, p=\frac{1}{3}, q=\frac{5}{3}.$

$y(-0.05)=0, y'(-0.05)=-1.81, n=1, \mu=1,$
$\varepsilon=0.01, h(t)=0.25t^3, p=1, q=1.$

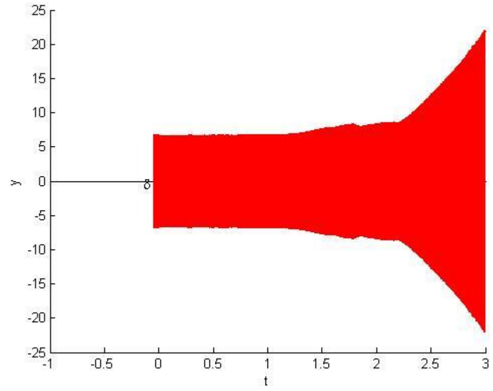

Fig. 7 *ode45: Numerical solution of (1), (2),*
$y(-0.05)=0, y'(-0.05)=-1.81, n=1, \mu=1,$
$\varepsilon=0.01, h(t)=0.25t^3, p=2, q=0.$

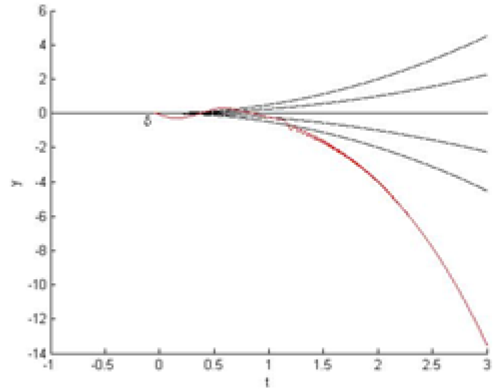

Fig. 8 *ode23: Numerical solution of (1), (2),*
$y(-0.05)=0, y'(-0.05)=-1.81, n=1, \mu=1,$
$\varepsilon=0.01, h(t)=0.25t^3, p=0, q=2.$

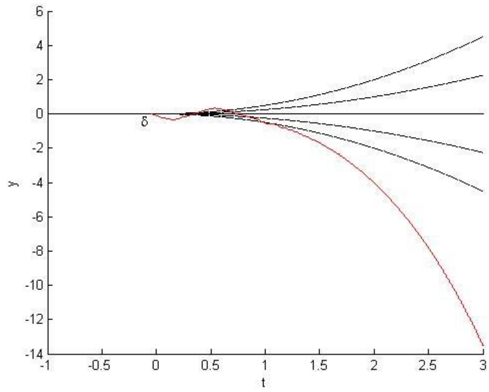

Fig. 9 *ode23: Numerical solution of (1), (2),*
$y(-0.05)=0, y'(-0.05)=-1.81, n=1, \mu=1,$
$\varepsilon=0.01, h(t)=0.25t^3, p=\frac{1}{40}, q=\frac{79}{40}.$

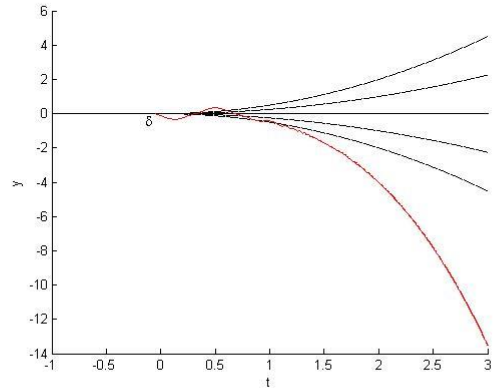

Fig. 10 *ode23: Numerical solution of (1), (2),*
$y(-0.05)=0, y'(-0.05)=-1.81, n=1, \mu=1,$
$\varepsilon=0.01, h(t)=0.25t^3, p=\frac{1}{20}, q=\frac{39}{20}.$

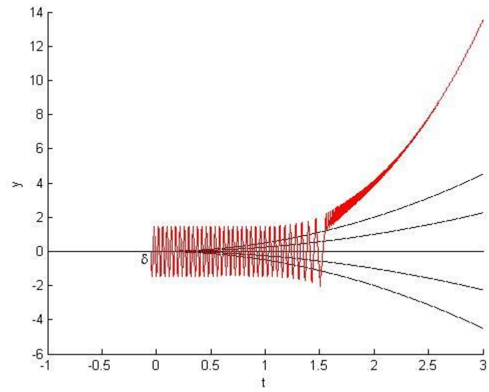

Fig. 11 *ode23: Numerical solution of (1), (2),*

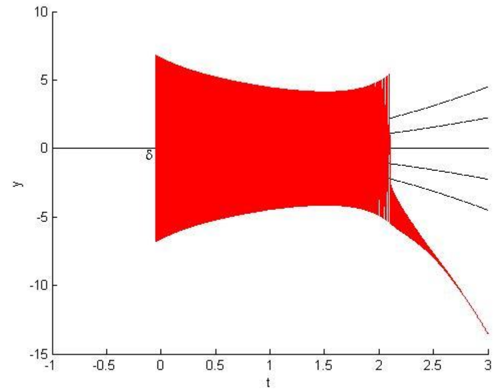

Fig. 12 *ode23: Numerical solution of (1), (2),*

$y(-0.05)=0,\ y'(-0.05)=-1.81,\ n=1,\ \mu=1,$
$\varepsilon=0.01,\ h(t)=0.25t^3,\ p=1,\ q=1.$

$y(-0.05)=0,\ y'(-0.05)=-1.81,\ n=1,\ \mu=1,$
$\varepsilon=0.01,\ h(t)=0.25t^3,\ p=2,\ q=0.$

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