RESEARCH PAPERS
FACULTY OF MATERIALS SCIENCE AND TECHNOLOGY IN TRNAVA
SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA

2016 Volume 24, Number 39

CONTRIBUTION TO DYNAMICAL PROPERTIES OF MACHINE AGGREGATES WITH GEARING

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Abstract

This contribution is focused on the impact of real gearing to the dynamics of gearing and a machine aggregate system. It is shown that a mechanical reduction gearbox is the source of excitation with a large range of frequencies. The mechanical part of drives, with respect to its electrical part, is the subject of control which should ensure optimal movement modes with regards to the technological process.

Key words

machine aggregate, gearing, dynamical model, transmission

INTRODUCTION

A rotary machine aggregate consists of a subsystem electric drive, which comprises an electric motor and transmission gear, a working machine which simulates some technological process, and a control assembly which ensures optimal control of the machine aggregate with respect to the technological process or dynamical properties of the aggregates (Fig.1).

A standard dynamometer is typically used to carry out static measurements of the machine aggregates and their subsystems further of the electric, internal-combustion, hydraulic and control drives, gearboxes, couplings, variators, and so on. The equipment makes it possible to measure static features of the above mentioned systems and their subsystems. The measurement is carried out point by point.

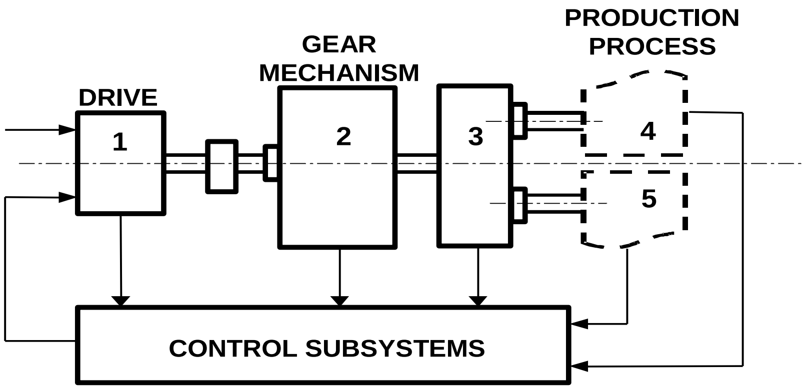


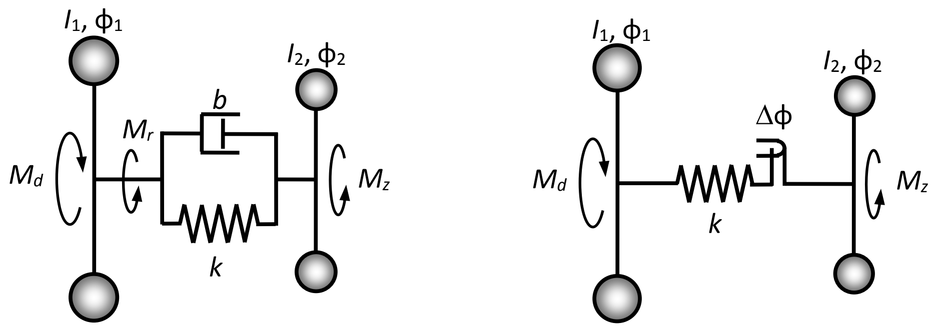
Fig. 1 *Scheme of machine aggregate*

Wear of the individual components of the machine aggregate and their progressive deformations, emergence of clearances in kinematical couplings and the like, are reasons resulting in modifications of the dynamical properties of the machines. The mentioned facts result in an increase of the energetic vibration level which brings an additional dynamical loading and decrease of reliability in machine operation when transmitting with them (5, 6).

DYNAMICAL MODEL OF MACHINE AGGREGATE WITH GEARING

In this paper, the problems of dynamical loads existing within the machine aggregate are analyzed, when elastic coupling, viscous damping, kinematical clearances and geometrical inaccuracies in gearing are taken into consideration. The influence of the dynamical load on the dynamical property of the aggregate as one unit is analyzed in this paper.

The dynamical model of the machine aggregate (Fig. 2a) is considered as a two-disc system. The first disc (moment of inertia I_1) is fixed on shaft 1 and consists of an electric motor and the gearing. The second disc (moment of inertia I_2) is fixed on shaft 2 and is considered as the effect of the working machine. The mutual joining of both shafts is represented by coupling the element with moment M_{kb}.



a) b)

Fig. 2 *Dynamical model of machine aggregate with gearing*
a - two-discs system with damping and without clearance,
b - two-discs elastic system with gearing clearance

The motion equations of the machine aggregate (Fig. 2b) with the gear drive and considered elastic coupling and viscous damping can be written as follows:

$$I_1 \frac{d\omega_1}{dt} = M_d - M_{kb} - M_r, \;\; I_2 \frac{d\omega_2}{dt} = M_{kb} - M_z, \qquad [1]$$

where

I_1, I_2 - the resulting reduced moments of inertia of the driving and driven part,
φ_1, φ_2- the angles of discs rotation,
ω_1, ω_2 - the angular velocities of discs,
M_d- driving (electro-magnetic) torque of the motor,
M_z- resulting reduced loading torque,
M_r- resistance torque, representing losses in the motor,
k - stiffness of the coupling element,
b - viscous damping coefficient of the coupling element,
$M_{kb} = M_k + M_b$ - torque transmitted by the coupling element,
$M_k = k(\varphi_1 - \varphi_2)$ - elastic coupling torque,
$M_b = b(\dot{\varphi}_1 - \dot{\varphi}_2)$ - coupling torque of viscous damping.

Resulting reduced gearing clearance can be expressed [1] by formula

$$\Delta\varphi = \sum_{i=1}^{n_r} \Delta\varphi_i \frac{\omega_1}{\omega_i} + \sum_{j=1}^{n_t} \Delta x_j \frac{\omega_1}{v_j}, \qquad [2]$$

where

$\Delta\varphi$ - resulting reduced clearance,
$\Delta\varphi_i$ - real gearing clearance of the i^{th} rotating member,
Δx_j - real clearance of the j^{th} translating member,
ω_1 - angular velocity of the main member,
ω_i - angular velocity of the i^{th} member,
v_j - translation velocity of the j^{th} member.

At first sight, it seems no excitation for the individual members is taken into consideration in the equations (1). As a consequence of the angular clearances, source of excitation is the torque of the elastic coupling M_k, which can be expressed in form

$$M_k = k(\varphi_1 - \varphi_2 - \Delta\varphi_{max} \sin \omega t), \qquad [3]$$

where $\Delta\varphi_{max}$ - maximal angular deviation within gearing reduced on the motor shaft, ω - angular frequency proportional to the angular rotor velocity of the drive.

The torque of the elastic coupling is then acting on both discs with angular frequency ω which is in the opposite phase to the angular speed of the drive. Gearing clearance $\Delta\varphi_i$ which depends on the gearing module and when a reduction on the rotor of the drive is made ($\Delta\varphi_i$ multiplied by ratio ω_1/ω_i), has a dominant influence on the resulting clearance (5).

Equations (1) can be written in the form

$$\ddot{\varphi} + \frac{I_1 + I_2}{I_1 I_2} b\dot{\varphi} + \frac{I_1 + I_2}{I_1 I_2} k\varphi = \frac{M_d - M_r}{I_1} + \frac{M_z}{I_2}, \qquad [4]$$

where $\varphi = \varphi_1 - \varphi_2$ is a relative angular deviation.

After arrangement, the equation (4) has the general form

$$\ddot{\varphi} + 2\zeta\omega_0\dot{\varphi} + \omega_0^2\varphi = f(t)\,, \qquad [5]$$

where

$\omega_0 = \sqrt{k\dfrac{I_1 + I_2}{I_1 I_2}}$ - natural angular frequency of undamped oscillations,

$\zeta = \dfrac{b}{2}\sqrt{\dfrac{I_1 + I_2}{k I_1 I_2}}$ - damping ratio,

$f(t) = \dfrac{\omega_0^2}{k}(I_2\varepsilon_\Phi + M_z)$ - right side of equation (5),

$\varepsilon_\Phi = \dfrac{M_d - M_r - M_z}{I_1 + I_2}$ - mean value of angular acceleration.

The equation (5) is a nonhomogeneous linear differential equation with constant coefficients and its general solution for can be written in the form (3)

$$\varphi(t) = \varphi_h(t) + \varphi_p(t) = C_h e^{-\zeta\omega_0 t}\sin(\omega_d t + \psi_h) + \frac{1}{\omega_d}\int_0^t f(\tau)e^{-\zeta\omega_0(t-\tau)}\sin(\omega_d(t-\tau))d\tau, \qquad (6)$$

where

$\omega_d = \omega_0\sqrt{1-\zeta^2}$ - natural angular frequency of the damped system,

C_h - amplitude of free vibration,

ψ_h - phase shift.

The resulting motion consists of a free vibration $(\varphi_h(t))$ of the system, which disappears after some time and after forced vibration $(\varphi_p(t))$. The solution depends on the form of function $f(t)$, i.e. on the manner of the load. After expression of $\varphi(t)$ and $\dot{\varphi}(t)$ from (6), the torque M_{kb} is transmitted by a binding element, which can be determined by:

$$M_{kb}(t) = k\varphi(t) + b\dot{\varphi}(t)\,. \qquad [7]$$

ANALYSIS OF LOADING WITHIN MACHINE AGGREGATE

In the following section it is shown, how the loading character is changed during start-up of the electromotor when the following parameters of the aggregate remain constant, i.e.

$$M_d = const.,\quad M_r = const.,\quad M_z = const., \qquad [8]$$

and ultimate stiffness is k and the damping is neglected ($b = 0$).

The equation of motion (4) can be modified into a form containing torque M_k of the elastic coupling $(M_{kb} = M_k,\ M_b = 0)$

$$\frac{1}{\omega_0^2}\ddot{M}_k + M_k = I_2\varepsilon_\Phi + M_z\,. \qquad [9]$$

The solution of the equation (9) for initial conditions

$$t = 0\,s: \qquad M_k(t)\big|_{t=0} = k\varphi(t)\big|_{t=0} = M_z\,,\quad \dot{M}_k(t)\big|_{t=0} = k\dot{\varphi}(t)\big|_{t=0} = 0$$

has the form

$$M_k(t) = M_z + I_2\varepsilon_\Phi(1-\cos(\omega_0 t))\,. \qquad [10]$$

The expression (9) can be expressed in the form

$$M_k(t) = M_{k,\Phi} - M_{k,a}\cos(\omega_0 t)\,, \qquad [11]$$

where $M_{k,\Phi} = M_z + I_2\varepsilon_\Phi$ is a mean value of real loading, $M_{k,a} = I_2\varepsilon_\Phi$ is an amplitude of variable component of the real loading.

From the expression [11] it follows, that the actual transmitted torque is a periodic function and it causes mechanical vibration in the system. These vibrations result in shocks causing the maximum value M_{kmax} of the real loading of the gearing to be higher than the mean value of the loading $M_{k\Phi}$ (3).

This state can be characterized by the so-called dynamical factor defined as the ratio of maximum loading value to mean loading value:

$$K_d = \frac{M_{k,\max}}{M_{k\Phi}} = \frac{M_z + 2I_2\varepsilon_\Phi}{M_z + I_2\varepsilon_\Phi}\,. \qquad [12]$$

From the relation [12] it follows, that the value of the dynamical factor rises with rising value ε_Φ and the value of the moment of inertia I_2. For special cases, the dynamical factor can achieve the value of 2, which means that the actual loading of the gearing is twice as high as the mean value of the actual loading.

CONCLUSION

The clearances [2] in the kinematical couplings and in gearing cause transitional effects in the aggregate. During free running time (disconnection of teeth contact) there is no mechanical coupling between bodies I_1 and I_2. For the simplest case (constant parameters of the machine aggregate), the body I_1 performs a uniformly accelerated rotating motion (acceleration ε_1 is constant) with angular speed (5).

During time t, the body I_2 is without motion or it is in uniform motion. When the clearance is overcome, the elastic impact between teeth occurs. The accumulated kinetic energy is changed into elastic deformation of teeth and into heat. It consequence of this, the growth of the torque transmitted by elastic coupling between aggregate members is observed and it brings the increase of the gearing dynamic loading.

If the initial condition (t = 0 s) is considered for the moment when gearing clearance is overcome, this transitional mode can be characterized as system starting. Now the dynamical model of the machine aggregate according to Fig.1b is taken into consideration. The clearances [2] existing in the kinematical couplings and within gearing are taken into consideration, from the equation (9) and for the following initial conditions.

Then the dynamical factor of the system with gearing clearance is given by expression:

$$K_d = 1 + \sqrt{1 + 2k\frac{I_2}{I_1 + I_2}\frac{M_d - M_r}{M_{k,\Phi}^2}\Delta\varphi} \qquad [13]$$

From the expression [13] results, that with given inertia moment I_1 of the motor, given reduced clearance $\Delta\varphi$, the dynamical loading of the gearing depends on acceleration at the moment of clearance overcoming and in relation to inertia moments of the drive and driven side of the machine. From this, it follows that the real gearing loading can be many times higher than the static one.

The dynamical factor is an important dynamical characteristic of the machine aggregate. From the analysis results, it is evident that the influence of the additional dynamical loadings has to be taken into consideration when the moment of inertia of the driven mechanism (machine) is much bigger than that of the motor. This reality has to be taken into consideration in calculations regarding motor power. In the cases when the inertia moment I_1 of the motor is dominant, the dynamical torque of the motor is spent on the acceleration of masses being firmly connected with the motor shaft. The transmissions of the machine are loaded by loading torque M_z only. This contribution is focused on the analysis of the influence of kinematical and geometrical deviations within gearing upon the dynamical properties of the machine aggregates. We showed that the mechanical gearbox of the machine aggregate is a source of excitation having a broad range of frequencies. The mechanical part of the drive becomes - in relation to its electric part - an object of regulation, which should ensure optimal regimes of system movement with regard to the expected technological process.

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