RESEARCH PAPERS
FACULTY OF MATERIALS SCIENCE AND TECHNOLOGY IN TRNAVA
SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA

2015 Volume 23, Special Number

CHARGED PARTICLE DYNAMICS INSIDE ELECTROSTATIC QUADRUPOLE DEFLECTION SYSTEM WITH CIRCULAR ELECTRODES AND ROTATIONALLY SYMMETRIC DESIGN

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Abstract

This paper offers a theoretical study of special type of electrostatic quadrupole deflection system (EQDS) intended for ion beam optics. We deal with EQDS consisting of electrode pairs with rotational symmetry design. This systems was pre-designed for an ion beam modification and trajectory controlling. Basic assumptions for determination of transfer characteristics of such systems are analysed on the basic of charged particle dynamics.

We are especially interested in the electrostatic field distribution among electrodes inside the mentioned type of EQDS. Typical case of the Sturm-Liouville boundary value problem called Bessel`s differential equation arises in calculation of the electrostatic scalar potential with rotational symmetry. Bessel`s functions are particular solution of Laplace equation in this case. The scalar potential equations of motion for ions in this electrostatic field are found. The path of charge-particles in this field could be determined by solving the trajectory equation of motion in Cartesian coordinates.

Key words

ion beam, quadrupole, Laplace equation, Bessel`s differential equation, Bessel`s functions,

1. INTRODUCTION

There are numerous types of electrostatic EQDS, each one of them is classified according to the properties and domain of application. In recent years, a number of publications have been devoted to the optimum design of EQDS and make the comparison between different electrodes shapes which are computed with the aid of transfer matrices. Design of such electrostatic quadrupole lens consists of four identical symmetric elements. The four-element lens systems have been widely investigated experimentally (1) and theoretically (2), showing that four element lenses are necessary to produce an image at a specific position and magnification. The ideal ESQ lens system consists of four parallel electrodes with hyperbolic cross-sections.

This research is concerned with the theoretical analysis of ion beam dynamics in electrostatic field inside a special type of ESQ lens with symmetrically placed flat circular-shaped electrodes. In most cases, electrostatic field distribution as well as ion beam trajectory inside electrostatic lenses (e.g. lenses included into an ion beam-lines) are modelled numerically, since analytical solutions are too complicated or even impossible. Numerical modelling is usually carried out using SIMION and LENSYS commercial packages, and a variety of performance parameters are obtained. In our contribution we consider lens consisting of electrically charged electrodes of circular shape generating electrostatic field, distribution of which is analytically solvable. The considered ESQ consists of four flat cylindrical electrodes mounted in pairs (see Fig.1).

In each case, ESQ performance is governed by the electrostatic fields among electrodes (which are determined by Laplace's equation) and the dynamics of ion motion (which are governed by Newton's laws). Some general methods to solve the electrostatic potential distribution have been developed. The first of them is the Separation of Variables Method which is used to solve Laplace`s equation in case of cylindrical symmetry. In this method, the solution of mentioned equation is written as a multiplication of functions where each of them depends on one variable only (3-10). With the Boundary Element Method (BEM) or Charge Density Method (CDM), the system of electrostatic lenses is replaced under the applied potentials by a system of rings of charge that assumes the same geometry as the cylinders (10-15).

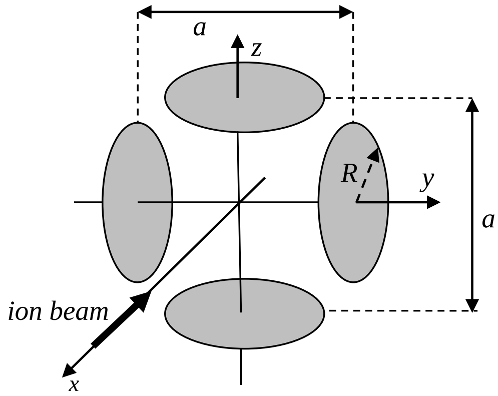

Fig. 1 *EQDS with rotationally symetric design*

A widely used method to obtain the electrostatic field distribution is The Finite Element Method (FEM). It is a numerical technique for obtaining solutions to boundary value problems. The principle of the method is that the potential distribution will be such that the potential energy of the electrostatic field is a minimum and so this potential energy is expressed in terms of the potentials at the vertices of each and all the triangular elements (16-18). A solution of Laplace`s equation can be also found by the Finite Difference Method (FDM). Among the most often used are the five-point and nine-point relaxation techniques (19-22).

We used the Separation of Variables Method and found equations of motion for ions passing through electrostatic field in special type of *EQDS* with rotationally symmetrical (flat circular-shaped) electrodes, and next we simplified those equations for the limit case of very narrow ion beam entering to the centre of the system. Solution of the mentioned equations will allow find ion beams trajectories and characterize considered system as an ion optical device. Our results provide the basis for analytical derivation of transfer characteristics (transfer matrices) for such specifically designed system as well. The motivation of the work is to investigate limitations for analytical study in some cases of specific shaped electrodes, and to study the possibility of generalizing of obtained results alternatively.

The novelty in this paper is limited, since charged segments of these types have been used occasionally since the early days of ion beam facilities. However, there may be a scope for accurate analytical solution of these structures using standard the technique of calculation that has not been considered to be important by earlier workers.

2. ELECTROSTATIC POTENTIAL BETWEEN A PAIR OF FLAT CIRCULAR POLES

Principles of electrostatic deflection systems are well understood. Here we give only a brief theoretical considerations relevant to the system with specific shape described above and based on standard procedures (23, 24). In the absence of charges, the electrostatic potential $\varphi(x;y;z)$ satisfies Laplace's equation:

$$\Delta\varphi = 0, \tag{1}$$

In case of rotational symmetry (see Fig.2), it is appropriate to transform equation [1] to cylindrical coordinates:

$$\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \theta^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0 \tag{2}$$

and consider the fact:

$$\frac{\partial^2 \varphi}{\partial \theta^2} = 0. \tag{3}$$

Consequently, the scalar potential $\varphi(r;z)$ can be found by solution of the following equation:

$$\frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{\partial^2 \varphi}{\partial z^2} = 0. \tag{4}$$

In cylindrical coordinates, the scalar potential is separable and it is given by the formula:

$$\varphi(r;z) = \Phi(r)\,Y(z). \tag{5}$$

Functions $\Phi(r)$ and $Y(z)$ must obey:

$$\frac{\partial^2 \Phi(r)}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi(r)}{\partial r} = \lambda\Phi(r), \tag{6}$$

$$\frac{\partial^2 Y(z)}{\partial z^2} = -\lambda Y(z). \tag{7}$$

Variation of the parameter λ gives different modes of potential distribution. λ is constant that can be positive, negative or it can be equal to zero. In the following text, we considered all mentioned events.

a) In the first case if $\lambda = 0$, the solution of equation [6] can be found by means of simple substitution:

$$f(r) = \frac{\partial \Phi(r)}{\partial r}. \tag{8}$$

As it can be easy shown:

$$f(r) = e^{-C_0}e^{-\ln(r)} = \frac{A_0}{r} \quad \text{and:} \quad \Phi(r) = A_0 \ln r + B_0. \tag{9}$$

Integration constants A_0 a B_0 depend on boundary conditions. Function $Y(z)$ in case if $\lambda = 0$ can be determined from equation [7]:

$$Y_0(z) = C_0 z + D_0, \tag{10}$$

where C_0 and D_0 are integration constants. Consequently in mentioned case ($\lambda = 0$) the particular solution of differential equation (4) can be written in the following form:

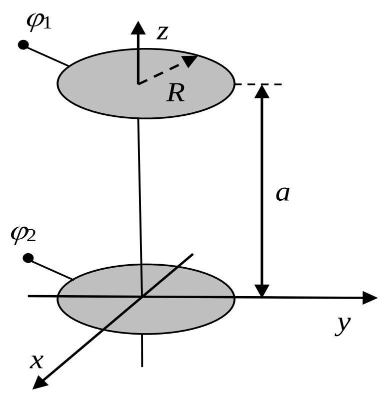

Fig. 2 *Pair of circular-shaped electrodes*

$$\varphi_0(r;z) = K_0 z \ln r + L_0 \ln r + M_0 z + N_0. \tag{11}$$

Integration constants K_0, L_0, M_0 a N_0 must be determined from boundary conditions.

b) In the second case if $\lambda > 0$, the λ can be written in the form:

$$\lambda = k^2. \tag{12}$$

Solution of equation [7] can be found as:

$$Y_p(z) = Y_1 \sin(kz+\alpha), \tag{13}$$

in this case. Equation [6] can be written as a special case of modified Bessel`s differential equation:

$$x^2 \frac{\partial^2 \Phi}{\partial x^2} + x\frac{\partial \Phi}{\partial x} - x^2\Phi = 0, \tag{14}$$

where: $x = kr$. [15]

Next, solution of (6) can be determined as follows:

$$\Phi_1(r) = A_2\xi_0(kr) + B_2\theta_0(kr). \tag{16}$$

ξ_0 and θ_0 are modified Bessel`s functions of zero order:

$$\xi_0(x) = \sum_{m=0}^{\infty}\frac{1}{(m!)^2}\left(\frac{x}{2}\right)^{2m} \quad , \quad \theta_0(x) = \frac{2}{\pi}\left\{\left[\gamma + \ln\left(\frac{x}{2}\right)\right]\xi_0(x) + \sum_{m=1}^{\infty}\frac{(-1)^{m+1}H_m}{(m!)^2}\left(\frac{x}{2}\right)^{2m}\right\} \tag{17}$$

where $H_m = \frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\ldots+\frac{1}{2m}$ and γ is Euler-Mascheroni constant: $\gamma = \lim_{n\to\infty}\left(H_n - \ln(n)\right) \approx 0{,}5772$.

Therefore, general solution of equation (4) can be found as:

$$\varphi^{(1)}(r;z) = \left[A_2\xi_0(kr) + B_2\theta_0(kr)\right]\sin(kz+\alpha) + K_0 z\ln r + L_0\ln r + M_0 z + N_0 \tag{18}$$

in the case of the $\lambda > 0$ consequently.

c) In the third case $\lambda < 0$, i.e.:

$$\lambda = -k^2, \tag{19}$$

solution of equation [7] can be found as:

$$Y(x) = A_3e^{kz} + B_3e^{-kz}, \tag{20}$$

where A_3 a B_3 are integration constants. Equation (6) can be written as a follows:

$$x^2 \frac{\partial^2 \Phi}{\partial x^2} + x\frac{\partial \Phi}{\partial x} + x^2\Phi = 0. \tag{21}$$

Equation (21) is special case of Bessel`s differential equation and solution of [6] can be found in the following form:

$$\Phi_2(x) = K_2\Lambda_0(x) + M_2\Omega_0(x), \tag{22}$$

where:

$$\Lambda_0(x) = \sum_{m=0}^{\infty}\frac{(-1)^m}{(m!)^2}\left(\frac{x}{2}\right)^{2m}, \quad \Omega_0(x) = \frac{2}{\pi}\left[\Lambda_0(x)\left\{C + \ln\left(\frac{x}{2}\right)\right\} + \frac{2}{1}\Lambda_2(x) - \frac{2}{2}\Lambda_4(x) + \frac{2}{3}\Lambda_6(x) + \ldots\right]$$

are Bessel`s functions of zero order. Then general solution of equation (4) is:

$$\varphi^{(2)}(r;z) = \left[K_2\Lambda_0(kr) + M_2\Omega_0(kr)\right]\left(A_3e^{kz} + B_3e^{-kz}\right) + K_0 z\ln r + L_0\ln r + M_0 z + N_0 \tag{23}$$

in case of $\lambda < 0$.

3. APPLICATION OF BOUNDARY CONDITIONS

Spatial distribution of scalar potential φ inside the EQDS must obey equation [4]. As we have shown in the previous section, functions $\varphi^{(1)}$ and $\varphi^{(2)}$ determined by [18] and [23] satisfy this equation. Now we need to decide which of these two functions describes the scalar potential among circular poles of the EQDS. Boundary conditions need to be applied in this decision.

In the area $z \le 0 \le a$, scalar potential φ must obey the following conditions:

$$\varphi(r;0) = \varphi_1,\ \varphi(r;a) = \varphi_2 \quad . \tag{24}$$

where φ_1 and φ_2 are electrodes potentials. It can be easily shown that only solution type [18] can satisfy boundary conditions [24]. Solution [23] cannot meet these conditions in any case. Therefore, spatial variation of electrostatic potential in the investigated area can be written in the form [18], while the following conditions must be satisfied in addition:

$$\sin(ka+\alpha)=0\,,\quad \sin(k0+\alpha)=0\,. \qquad [25]$$

It follows from [25]:

$$ka+\alpha=n\pi\quad,\ k0+\alpha=m\pi\ , \qquad [26]$$

where n a m are integral numbers (0, ±1, ±2, ±3,...). Taking into account the fact that there are no reasons that any part of solution is periodical, it is possible to consider m = 0 and n = 1.

Then: $\alpha=0\ ,\ k=\dfrac{\pi}{a}\,.$ [27]

In addition, the boundary conditions [24] can be satisfied only in case:

$$K_0=0\ ,\ L_0=0\,. \qquad [28]$$

For that reason, the scalar potential in the mentioned area must be written in following form:

$$\varphi(r;z)=\left[A_2\,\xi_0\left(\frac{\pi}{a}r\right)+B_2\,\theta_0\left(\frac{\pi}{a}r\right)\right]\sin\left(\frac{\pi}{a}z\right)+M_0\,z+N_0\ . \qquad [29]$$

After substituting [24] to [28], integration constants can be determined as:

$$N_0=\varphi_1\ ,\quad M_0=\frac{\varphi_2-\varphi_1}{a}=\frac{U}{a}\,. \qquad [30]$$

and formula for the scalar electrostatic potential takes the next form:

$$\varphi(r;z)=\left[A_2\,\xi_0\left(\frac{\pi}{a}r\right)+B_2\,\theta_0\left(\frac{\pi}{a}r\right)\right]\sin\left(\frac{\pi}{a}z\right)+\frac{U}{a}z+\varphi_1\,, \qquad [31]$$

where $U = \varphi_2 - \varphi_1$ is voltage applied to electrodes. For the intensity of the electrostatic field, it applies:

$$\vec{E}=-\nabla\varphi\,. \qquad [32]$$

In case of axial symmetry in cylindrical coordinates (considering [3]), it holds:

$$\vec{E}=-\frac{\partial\varphi}{\partial r}\vec{\rho}-\frac{\partial\varphi}{\partial z}\vec{k}=-E_{\perp}\vec{\rho}-E_{\parallel}\vec{k}\,, \qquad [33]$$

where $\vec{k}$ and $\vec{\rho}$ are unit vectors perpendicular and parallel to the electrode surface. As it is clear from the symmetry, in limit case:

$$\lim_{r\to0}E_{\perp}=0\,. \qquad [34]$$

From [17], it results that condition [34] can be satisfied only when B_2 = 0. Then the scalar potential φ and intensity $\vec{E}$ of electrostatic field between a pair of flat circular electrodes inside EQDS can be determined by the next formulas:

$$\varphi(r;z)=A\sum_{m=0}^{\infty}\frac{1}{(m!)^2}\left(\frac{\pi}{2a}r\right)^{2m}\sin\left(\frac{\pi}{a}z\right)+\frac{U}{a}z+\varphi_1\ , \qquad [35]$$

$$\vec{E}=-A\frac{\pi}{h}\sum_{m=1}^{\infty}\frac{1}{m!(m-1)!}\left(\frac{\pi}{2a}r\right)^{2m-1}\sin\left(\frac{\pi}{a}z\right)\vec{\rho}-\left\{A\frac{\pi}{a}\sum_{m=0}^{\infty}\frac{1}{(m!)^2}\left(\frac{\pi}{2a}r\right)^{2m}\cos\left(\frac{\pi}{a}z\right)+\frac{U}{a}\right\}\vec{k}\quad. \qquad [36]$$

Constant A is a free parameter enabling to take into account physical characteristics of the system.

4. SUPERPOSITION PRINCIPLE APPLICATION

Considered EQDS consists of two pairs of electrodes and each of this pairs generates electrostatic field described by formulas [35] and [36] respectively. The mentioned pairs are placed symmetrically and rotated by 90 degrees (see Fig.1). We note that adding the second pair of electrodes causes a symmetry breaking. Electrostatic field generated by the whole system is not rotationally invariant and problems with application of boundary conditions arise when applying the method described above. To avoid these problems we assume that a is larger than electrodes radius R, i.e. none of electrode lies in the calculated electrostatic field. In that case we assume that superposition principle can be applied in the estimation of entire field in the area between both pair of electrodes. In system configuration shown in Fig.1, distributions of electrostatic fields generated by both individual pairs of electrodes can be written by means of [36] as:

$$\vec{E}^{(1)} = -A\frac{\pi}{h}\sum_{m=1}^{\infty}\frac{1}{m!(m-1)!}\left(\frac{\pi}{2a}r\right)^{2m-1}\sin\left[\frac{\pi}{a}\left(z+\frac{a}{2}\right)\right]\vec{\rho} - \left\{A\frac{\pi}{a}\sum_{m=0}^{\infty}\frac{1}{(m!)^2}\left(\frac{\pi}{2a}r\right)^{2m}\cos\left[\frac{\pi}{a}\left(z+\frac{a}{2}\right)\right] + \frac{U_1}{a}\right\}\vec{k} \quad [37]$$

$$\vec{E}^{(2)} = -A'\frac{\pi}{h}\sum_{m=1}^{\infty}\frac{1}{m!(m-1)!}\left(\frac{\pi}{2a}r'\right)^{2m-1}\sin\left[\frac{\pi}{a}\left(y+\frac{a}{2}\right)\right]\vec{\rho} - \left\{A'\frac{\pi}{a}\sum_{m=0}^{\infty}\frac{1}{(m!)^2}\left(\frac{\pi}{2a}r'\right)^{2m}\cos\left[\frac{\pi}{a}\left(y+\frac{a}{2}\right)\right] + \frac{U_2}{a}\right\}\vec{k} \quad [38]$$

where $r = \sqrt{x^2+y^2}$, $r' = \sqrt{x^2+z^2}$ and U_1, U_2 are voltages applied on single pairs of electrodes. Next, we can consider:

$$\frac{\partial r'}{\partial x} = \frac{x}{r'} \quad , \quad \frac{\partial r'}{\partial z} = \frac{z}{r'} \qquad [39]$$

and transform formulas [37] and [38] to Cartesian coordinates. If we apply superposition principle given by well-known formula:

$$\vec{E} = \vec{E}^{(1)} + \vec{E}^{(2)} \qquad [40]$$

coordinates of intensity of total electrostatic field in the considered area inside the EQDS can be approximated as:

$$E_x = -\frac{\pi}{a}\sum_{m=1}^{\infty}\frac{1}{(m!)^2}\left(\frac{\pi}{2a}\right)^{2m-1} mx\left\{A\left(r^2\right)^{m-1}\cos\left(\frac{\pi}{a}z\right) + A'\left(r'^2\right)^{m-1}\cos\left(\frac{\pi}{a}y\right)\right\} \qquad [41]$$

$$E_y = -\frac{\pi}{a}\sum_{m=1}^{\infty}\frac{1}{(m!)^2}\left(\frac{\pi}{2a}\right)^{2m-1}\left\{Amy\left(r^2\right)^{m-1}\cos\left(\frac{\pi}{a}z\right) - A'\left(r'^2\right)^{m}\left(\frac{\pi}{2a}\right)\sin\left(\frac{\pi}{a}y\right)\right\} + A'\frac{\pi}{a}\sin\left(\frac{\pi}{a}y\right) - \frac{U^{(2)}}{a} \qquad [42]$$

$$E_z = -\frac{\pi}{a}\sum_{m=1}^{\infty}\frac{1}{(m!)^2}\left(\frac{\pi}{2a}\right)^{2m-1}\left\{-A\left(r^2\right)^{m}\left(\frac{\pi}{2a}\right)\sin\left(\frac{\pi}{a}z\right) + A'zm\left(r'^2\right)^{m-1}\cos\left(\frac{\pi}{a}y\right)\right\} + A\frac{\pi}{a}\sin\left(\frac{\pi}{a}z\right) - \frac{U^{(1)}}{a} \qquad [43]$$

where we used:

$$\cos\left(\alpha+\frac{\pi}{2}\right) = -\sin(\alpha), \quad \sin\left(\alpha+\frac{\pi}{2}\right) = \cos(\alpha).$$

It is clear from [41], [42] and [43] that intensity vector inside the EQDS can be written in the form:

$$\vec{E} = \vec{\eta} + \vec{\delta} \quad , \qquad [44]$$

where:

$$\vec{\eta} = -\left[0; \frac{U^{(2)}}{a}; \frac{U^{(1)}}{a}\right] \quad \text{and} \quad \vec{\delta} = -\frac{\pi}{a}\sum_{m=1}^{\infty}\left\{\frac{1}{(m!)^2}\left(\frac{\pi}{2a}\right)^{2m-1}\left(\vec{\varepsilon}_m + \vec{\mu}_m\right)\right\} - \vec{\nu}\,. \qquad [45a,b]$$

$\vec{\eta}$ is a component corresponding to the homogeneous electrostatic field in ideal case of infinitely large electrodes and $\vec{\delta}$ is correction to the field scattering generated on edges of electrodes. Individual vectors in formula [46b] are:

$$\vec{\varepsilon}_m = A\left(x^2+y^2\right)^{m-1}\left[mx\cos\left(\frac{\pi}{a}z\right); my\cos\left(\frac{\pi}{a}z\right); -\left(x^2+y^2\right)\left(\frac{\pi}{2a}\right)\sin\left(\frac{\pi}{a}z\right)\right] \qquad [46]$$

$$\vec{\mu}_m = A'\left\{x^2+z^2\right\}^{m-1}\left[mx\cos\left(\frac{\pi}{a}y\right); -\left\{x^2+z^2\right\}\left(\frac{\pi}{2a}\right)\sin\left(\frac{\pi}{a}y\right); mz\cos\left(\frac{\pi}{a}y\right)\right] \qquad [47]$$

$$\vec{\nu} = \frac{\pi}{a}\left[0; -A'\sin\left(\frac{\pi}{a}y\right); -A\sin\left(\frac{\pi}{a}z\right)\right]. \qquad [48]$$

5. EQUATIONS OF MOTION

Equations of motion of ion with charge q and mass M in non-relativistic case can be written as:

$$M\frac{d^2\vec{r}}{dt^2}=q\vec{E}\,. \qquad [49]$$

where $\vec{r}=[x,y,z]$ is an ion position vector. It is necessary to substitute the calculated intensity vector [44] to equation [49]. However, expression [49] is then quite complicated whereas it contains sums [45b]. But if the ion beam is very thin and it spreads around x axis (i.e. $z \ll a$ and $y \ll a$), the next approximation is possible:

$$\frac{\pi}{a}z\to 0\ ,\ \ \frac{\pi}{a}y\to 0\ ,\ \text{and therefore}\ \sin\left(\frac{\pi}{a}z\right)\approx\frac{\pi}{a}z\,,\ \cos\left(\frac{\pi}{a}z\right)\approx 1\,. \qquad [50]$$

Taking into account [50] equations of motion can be simplified to the form:

$$M\frac{d^2x}{dt^2}=-q\frac{\pi}{a}x\sum_{m=1}^{\infty}\frac{1}{(m!)^2}\left(\frac{\pi}{2a}\right)^{2m-1}m\left\{A\left(x^2+y^2\right)^{m-1}+A'\left(x^2+z^2\right)^{m-1}\right\} \qquad [51]$$

$$M\frac{d^2y}{dt^2}=-\frac{qU_2}{a}-q\frac{\pi}{a}y\sum_{m=1}^{\infty}\frac{1}{(m!)^2}\left(\frac{\pi}{2a}\right)^{2m-1}\left\{Am\left(x^2+y^2\right)^{m-1}-A'\frac{1}{2}\left(\frac{\pi}{a}\right)^2\left(x^2+z^2\right)^{m}\right\}+A'q\left(\frac{\pi}{a}\right)^2\left(\frac{y}{2}\right) \qquad [52]$$

$$M\frac{d^2z}{dt^2}=-\frac{qU_1}{a}-q\frac{\pi}{a}z\sum_{m=1}^{\infty}\frac{1}{(m!)^2}\left(\frac{\pi}{2a}\right)^{2m-1}\left\{-A\frac{1}{2}\left(\frac{\pi}{a}\right)^2\left(x^2+y^2\right)^{m}+A'm\left(x^2+z^2\right)^{m-1}\right\}+Aq\left(\frac{\pi}{a}\right)^2\left(\frac{z}{a}\right)\,. \qquad [53]$$

When we assume that squared coordinates are very small numbers, higher powers in series on the right sides of equations [51], [52] and [53] can be neglected. In general, it is sufficient to consider only the first members of series. However, in the application of that approximation, we must currently take into account that electrostatic intensity must hold:

$$\nabla\times\vec{E}=\vec{0}\,. \qquad [54]$$

To satisfy the condition [54], it is necessary to consider both first and second members of series in equation [51]. From [54], it also results:

$$A=A'$$

and the next form of equation of motion can be written in the limit case:

$$M\frac{d^2x}{dt^2}=-qA\left(\frac{\pi}{a}\right)^2\left\{1+\left(\frac{\pi}{2a}\right)^2\left[\left(x^2+y^2\right)+\left(x^2+z^2\right)\right]\right\}x \qquad [55]$$

$$M\frac{d^2y}{dt^2}=-\frac{qU_2}{a}+qA\left(\frac{\pi}{a}\right)^2\left\{1-\left(\frac{\pi}{2a}\right)^2\left(x^2+z^2\right)\right\}y \qquad [56]$$

$$M\frac{d^2z}{dt^2}=-\frac{qU_1}{a}+qA\left(\frac{\pi}{a}\right)^2\left\{1-\left(\frac{\pi}{2a}\right)^2\left(x^2+y^2\right)\right\}z\ . \qquad [57]$$

Parameter A can be determined by considering the limit values for $a\to\infty$, $R\to\infty$. On the basis of this, we believe that:

$$A=-\chi\left(\frac{a}{\pi^2R}\right)\sqrt{U_1^2+U_2^2}\ , \qquad [58]$$

where χ is the constant depending on ion dynamic characteristic (energy). Equations of motions [55], [56] and [57] must be solved if one is looking for trajectory of ion beam passing through the area inside the considered EQDS. For optimum performance, the design parameters of the EQDS a and R should be chosen. Ion enters the EQDS field on the x axis (in initial position of ion $y = 0$ and $z = 0$) and it is next deflected by the electrostatic force. From [56] and [57], it is clear that deflection of ion beam in directions of applied voltages (i.e. directions of both y and z axes) is governed by force components:

$$F_y=-\frac{qU_2}{a}+yf(x,z)\,,\ F_z=-\frac{qU_1}{a}+zf(x,y)\,, \qquad [59]$$

where: $$f(x, x_k) = qA\left(\frac{\pi}{a}\right)^2 \left\{1 - \left(\frac{\pi}{2a}\right)^2 \left(x^2 + x_k^2\right)\right\}, \quad x_k = y, z \ . \qquad [60]$$

Equations [59] reflect symmetry of EQDS design and represent the path to reveal the relationship between the dynamics of ion and optical properties of the EQDS.

6. CONCLUSION

Several types of deflection systems are used at present time, including the ring, conical, and hemispherical designs. The quadrupole system has many variable geometrical and operational parameters, thus conclusive result in this field is rather difficult. Our paper focuses on the system with special type of design characterized by rotational symmetry. The design of this EQDS was described above. We found how the scalar potential function is related to the system geometry in this case and reviewed the ion dynamics in such special type EQDS. The potential distribution inside the quadrupole system is calculated by the Separation of Variables Method solving analytically Laplace's equation in three dimensions.

Finally, we would like to point out that the presented contribution is only focused on preliminarily results. The results reported above allow us to start the discussion on the possibility to describe such systems by means of the methods used in ion beam optics. However, the practical application of such system design remains unclear yet and requires experimental observations.

Acknowledgement

This research was funded by the ERDF - Research and Development Operational Programme under the project of "University Scientific Park Campus MTF STU - CAMBO" ITMS: 26220220179.

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