THE EFFECT OF CRACK PARAMETERS ON MODAL PROPERTIES OF TUBE STRUCTURES

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Abstract

The appearance of cracks in the structures such as perforated tubes of the protective tube unit introduces local modification of flexibility and consequently affects the dynamic behaviour of the cracked structural member. The bending vibrations of cracked protective tube are studied in this paper. The influence of the fundamental crack geometrical parameters, especially the position and depth of the crack, on the modal properties (such as the natural frequencies and mode shapes) is examined. Determination of the required modal properties is done using ANSYS FEM computing system. Results of the change in modal parameters due to the crack appearance are presented in this paper.

Key words

vibration, modal properties, crack, perforated structure, finite element method

Introduction

The structural elements, such as tubes, cylindrical shells, pipe structures are very often used in the reactor technologies for nuclear industry. Special attention must be paid particularly to such structural element in the reactor system, the failure of which could cause potential problems leading to the loss of operational capability or the reactor accident.

One of such structural elements is a tube of the protective tube unit. During operation, these perforated tubes are exposed to different load cases, which are caused by the influence of the flow of the cooling medium, i.e. fluid-structure interaction. As a result of the flow of coolant, undesirable dynamic effects and overload protective tubes are occurring. Consequently, in the case of continuing action of these dynamic effects, the possibilities for the development of degenerative states in protective tubes are provided and these states may cause the initiation of structural defects such as cracks. The crack presence in the structure not only causes the local change in stiffness [4], but cracks have a significant effect on the mechanical behaviour of the whole structure. Cracks present in vibrating structures, as time progresses, could lead to the catastrophic failure and breakdown of the structure.

In order to improve the safety, reliability and operational life, it is important to ensure the integrity of structural elements. The cracks or other defects in a structural element influence its dynamical behaviour and change its stiffness and damping properties. The cracks in a vibrating structure reduce its natural frequencies because it becomes more flexible. The natural frequencies and mode shapes of the structure contain the information [1], [3] about the location and dimensions of the damage.

The dynamic behaviour of perforated protective tube can be considered the same dynamic behaviour as in the case of beam structure [2]. This approximation results from the high slenderness of protective tubes. As a representative model for cracked protective tubes, a clamped-clamped beam with a transverse open crack is used in this paper. The effect of two of the crack parameters on the modal properties (natural frequencies, mode shapes) of the cracked beam is studied. These two parameters are the depth and position of the crack.

Problem formulation

The problem considered here is a simple beam (tubes) with transversal crack as it is shown schematically in Fig. 1. The beam of length L_0 has a uniform cross-section (outer diameter - D_0, thickness - t_p). It is supposed that the beam has the crack of depth t_c located at a distance l_c from the right-side of the beam. Only a fully open crack is considered. The beam is clamped on both ends.

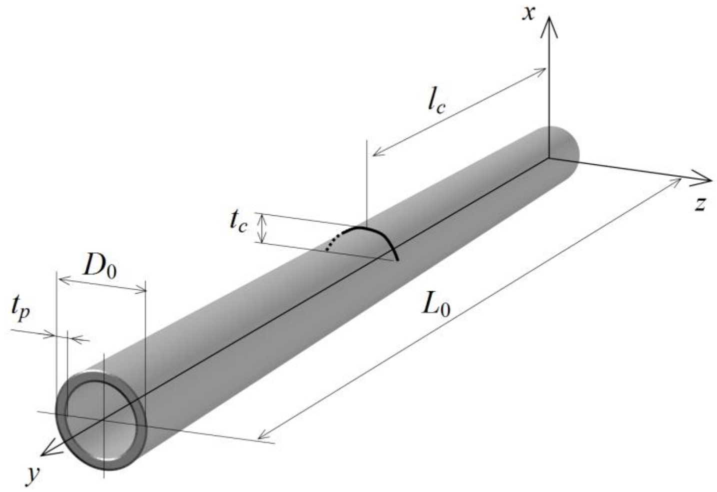


Fig.1 *Tubes with transversal crack*

The mathematical model used for modal analysis of this beam is based on the finite element method. The three-dimensional finite element model of the beam with transverse non-propagating crack to analyse the crack effect on the modal properties is used.

Generally, the equation of motion for a free vibration of beam without crack has the form

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0}, \qquad [1]$$

where $\mathbf{M}$ is a mass matrix, $\mathbf{K}$ is a stiffness matrix of the beam without crack, vectors $\mathbf{q}$ and $\ddot{\mathbf{q}}$ are displacements and accelerations, respectively.

The modal properties of a beam - natural angular frequencies and mode shapes, can be obtained from the solution if the following eigenvalue problem

$$(\mathbf{K} - \omega^2\mathbf{M})\boldsymbol{\phi} = \mathbf{0}, \qquad [2]$$

where ω is the natural angular frequency and $\boldsymbol{\omega}$ is the mode shape.

The birth and growth of the crack are affecting natural frequencies and mode shapes of cracked beam. It is clear that the mass distribution of cracked beam is not changed. The beam bending stiffness is significantly modified by the formation of crack, i.e. when the crack is considered, the stiffness matrix of the beam has to be changed.

The equation of motion for a free vibration of cracked beam has the form

$$\mathbf{M}\ddot{\mathbf{q}}_c + \mathbf{K}_c\mathbf{q}_c = \mathbf{0}, \tag{3}$$

Then, the eigenvalue problem for the cracked beam structure can be written as

$$(\mathbf{K}_c - \omega_c^2\mathbf{M})\boldsymbol{\phi}_c = \mathbf{0}, \tag{4}$$

where $\mathbf{K}_c$ is a stiffness matrix of the cracked beam, ω_c is a natural angular frequency and $\boldsymbol{\phi}_c$ is a mode shape vector of the cracked beam.

The modal analysis of a beam with crack was done using the ANSYS FEM code.

Numerical results

The effect of the transverse crack position and depth on the modal properties is investigated. The natural frequencies and mode shapes of the clamped-clamped (C-C) beam structure are studied.

The beams under analysis have the following properties: uniform piping cross-section with outer diameter D_0 =0.1 mm and thickness t_p = 0.01 mm. Slenderness of beam structure is expressed by

$$\delta = \frac{L_0}{D_0}, \tag{5}$$

and three values of beam slenderness are considered, i.e. $\delta = \{10; 15; 20\}$.

Material properties of the beam structure are: Young's modulus of elasticity E = 210 GPa, Poisson number μ = 0.3 and density ρ = 7800 kgm^3.

The geometrical parameters of the crack are specified by using the following dimensionless crack parameters:

- dimensionless crack position $$\delta_c = \frac{l_c}{L_0} \in \langle 0.0; 1.0\rangle, \tag{6}$$
- dimensionless crack depth $$\psi_c = \frac{t_c}{D_0} \in \langle 0.0; 0.8\rangle. \tag{7}$$

To generalise the results obtained, the dimensionless natural frequency for i-th mode shape is introduced and it is defined as a frequency ratio

$$\vartheta_i = \frac{f_{c,i}}{f_i}, \tag{8}$$

where f_i is i-th natural frequency of beam without crack and $f_{c,i}$ is i-th natural frequency of cracked beam structure.

The computational model of the cracked beam structure is shown in Fig.2a and finite element mesh structure is presented in Fig.2b. The first four typical modal shapes of vibration the considered beam structures (slenderness is greater than 10) are shown in Fig.2c ÷ Fig.2f. The existence of crack significantly affects the modal properties of the cracked beam structure. The main finding is the fact that the bending stiffness around the axes x and z are different due to the formation of cracks. In the case of the beam without a crack, first two and second two frequencies are the same. In the case of beam with a transverse crack, the natural frequencies for bending vibration around the z axis (i.e. the beam is deformed in the xy plane) are lower, which is caused by the smaller flexural stiffness. The corresponding mode shapes are mutually rotated by 90 degrees.

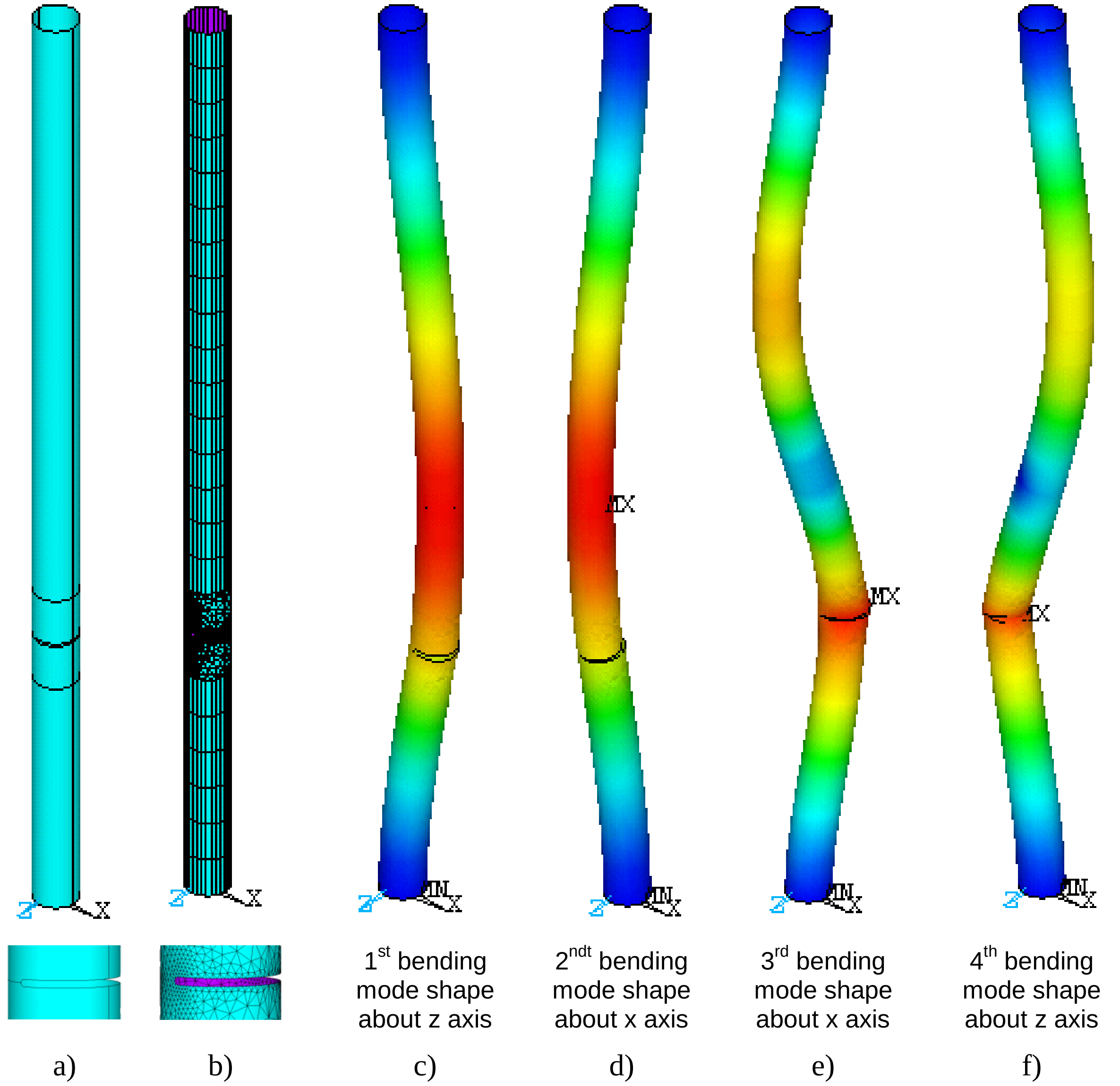


Fig. 2 *Dependency of the first four dimensionless natural frequencies* ϑ_i *C-C beam structure on dimensionless crack position* δ_c for different ψ_c (*slenderness* δ = 10).

The dependencies of the first four natural frequencies of C-C beam on the crack position δ_c for the different crack depths ψ_c are shown in Fig.3÷Fig.5 (for different values of slenderness). Depending on the location and depth of crack, the natural frequency for some combinations of crack parameters (δ_c and ψ_c) are reduced up to 40 per cent compared to the beam structure without crack. These curves of dimensionless frequencies (Fig.3÷Fig.5) have more local minima and maxima, respectively. The number of local extremes (maximum, minimum) of the curve depends on the natural frequency and on the corresponding mode shape.

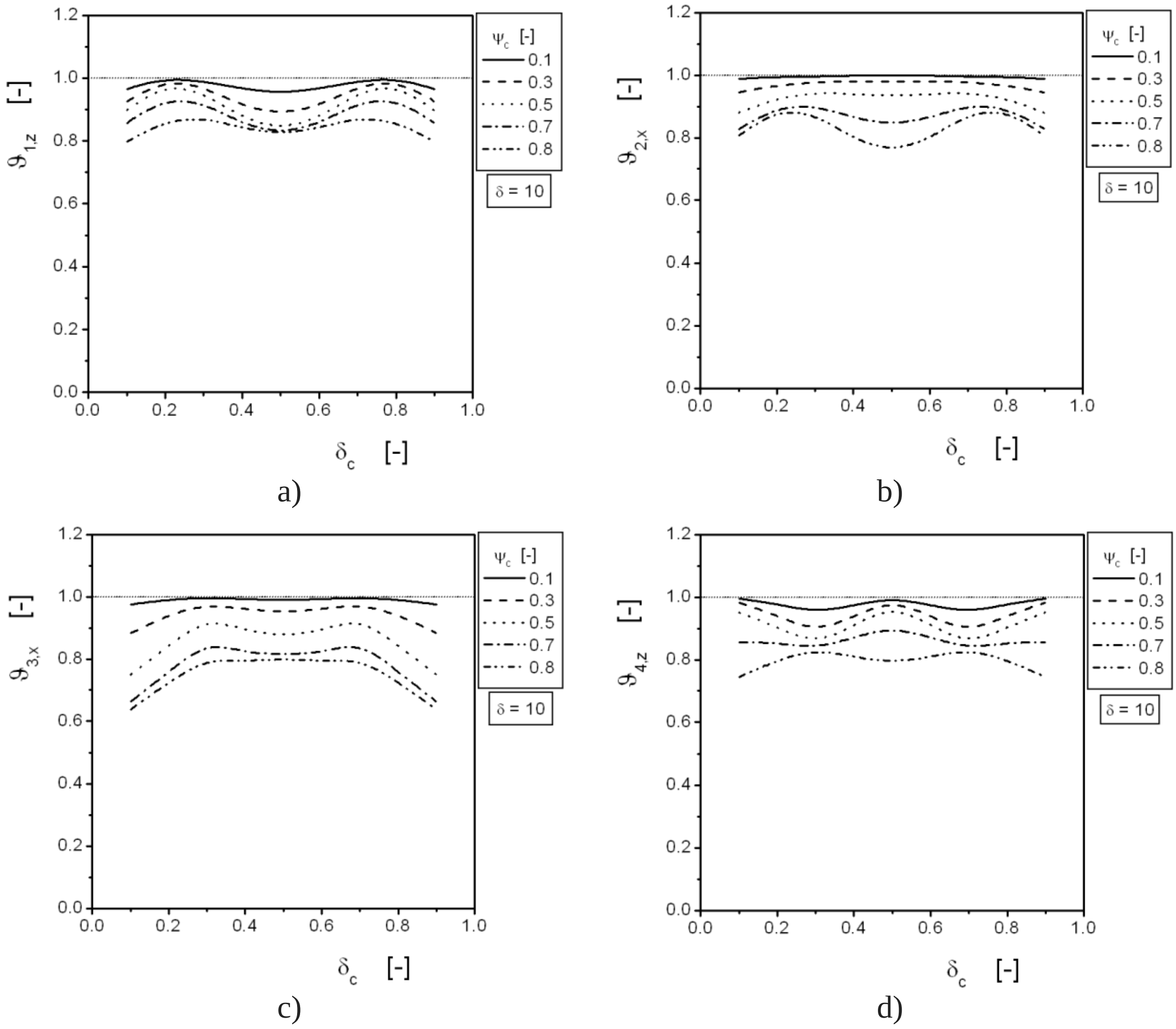
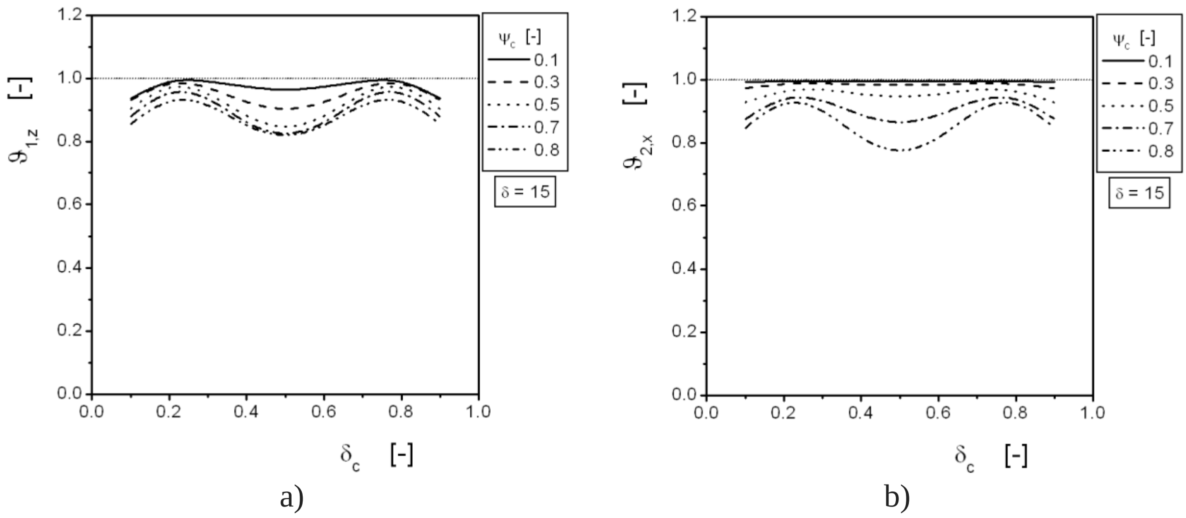


Fig. 3 *Dependency of the first four dimensionless natural frequencies* ϑ_i *C-C beam structure on dimensionless crack position* δ_c for different ψ_c (*slenderness* δ = 10).



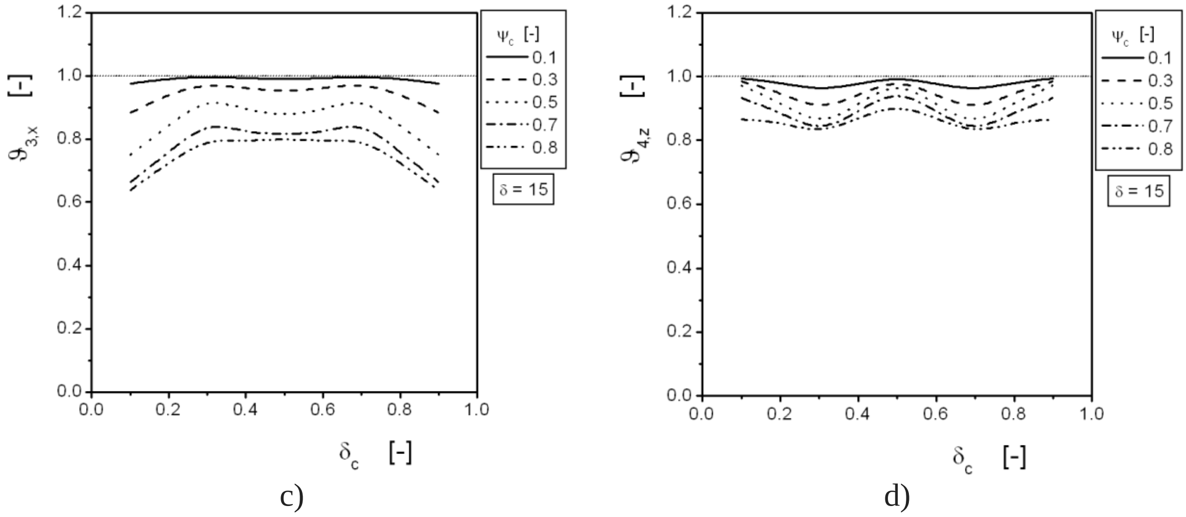


Fig. 4 *Dependency of the first four dimensionless natural frequencies* ϑ_i *C-C beam structure on dimensionless crack posit*ion δ_c for different ψ_c (*slenderness* δ = 15).

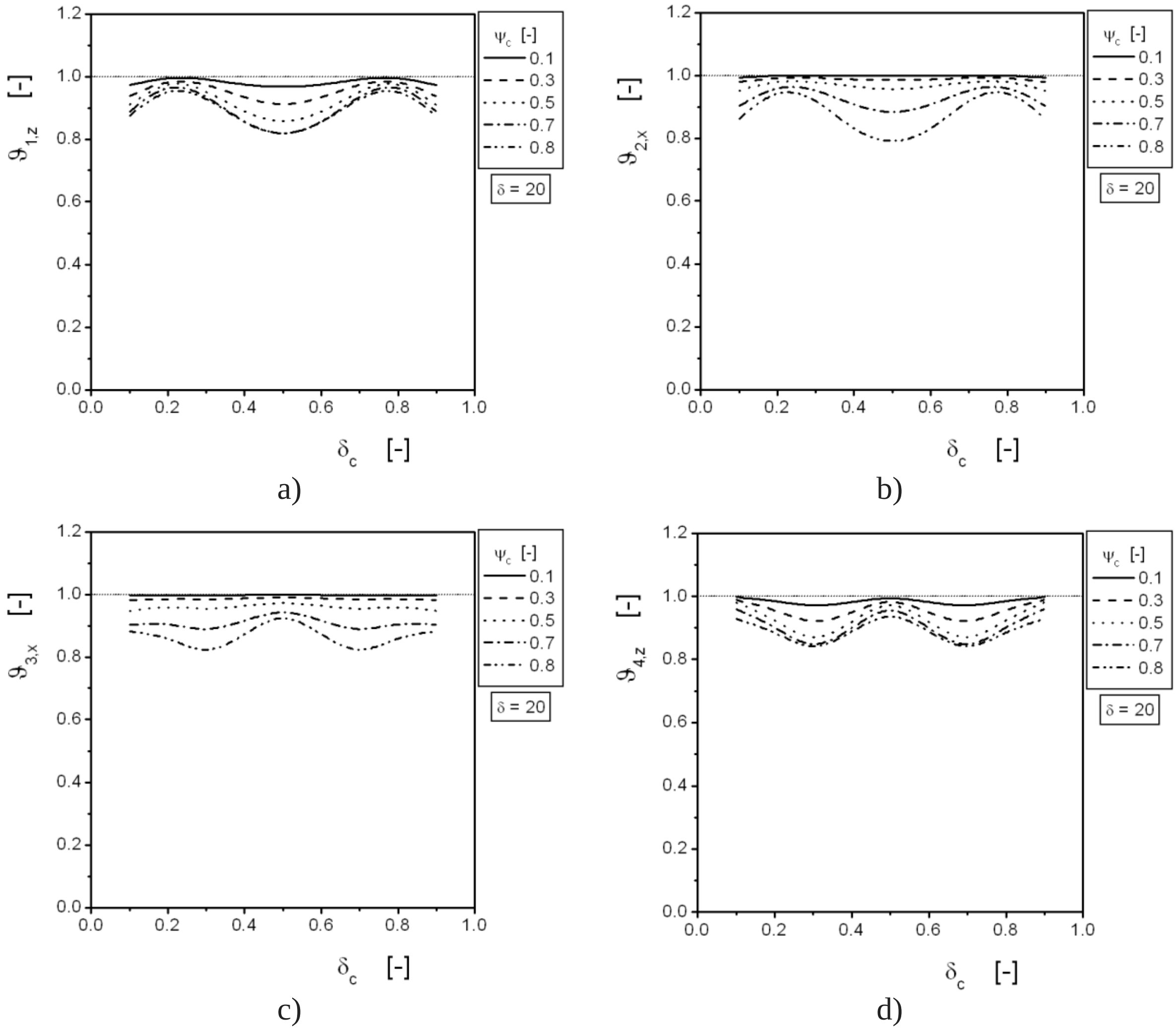


Fig. 5 *Dependency of the first four dimensionless natural frequencies* ϑ_i *C-C beam structure on dimensionless crack posit*ion δ_c for different ψ_c (*slenderness* δ = 20).

Conclusion

The effect geometrical parameters of transversal crack on natural frequencies and mode shapes C-C beam structure is analysed in the present paper. The crack depth and crack position cause a notable modification of natural frequencies and mode shapes of bending vibration of beams. The results of this study showed that, in some cases, the existence of a crack in the beam structure may reduce the natural frequencies up to 40 % compared to the volume structure without cracks. Regarding the operation of this structure with reduced value natural frequency (caused by the existence of crack), it is necessary to point out the possibility of resonant states. In such a structure, the resonance states can occur at lower excitation frequencies than those that were predicted for the undamaged structure.

Finally, it can be said that the crack is the damage occurring in the structural elements that can cause serious equipment failures, and may lead to its destruction. This suggests that the crack must be detected in the early state.

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