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GENERALIZED CONSTRUCTION OF TWO-DIMENSIONAL QUASI COMPLETE COMPLEMENTARY CODES

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ABSTRACT

Authors in [1] proposed generating method of two-dimensional (2D) quasi orthogonal complete complementary codes (QOCCC) with element order N^2. This construction is based on one-dimensional (1D) complete complementary codes (CCC) introduced in [2]. New constructions of CCC with shorter 1D-CCC element length were published recently [3-4]. In this paper former construction is generalized for using any recent 1D-CCC for input which results in 2D-CCC with element of smaller order. Parameter dependences of 2D-QOCCC on 1D-CCC parameters and advantages of QOCCC are discussed.

KEY WORDS

Quasi orthogonal complete complementary code, QOCCC, complete complementary code, CCC, two-dimensional.

INTRODUCTION

Complete complementary codes (CCC) found application in many different areas, such as physics, astronomy, radar technology, navigation, image processing and telecommunication systems. Golay in [5] published complementary codes in connection with optical infrared, multi-slit spectrometry first. Later further complementary sequences were developed in [6-9]. Authors of mentioned papers were concerned mostly about autocorrelation properties of those sequences. First who proposed sets of sequences with ideal autocorrelation and cross-correlation were Tseng and Liu [10]. Later these sequence sets were called Complete Complementary Codes (CCC) [7-9]. Generating of CCCs is still a vivid area of research. Latest framework for a systematic construction was published recently [4].

In [1] authors proposed construction of two-dimensional quasi orthogonal complete complementary codes (2D-QOCCC) with element order of N^2 where on input 1D-CCC [2] with sequence length of N^2 was used. In this paper original method is generalized to accept on input any of recent 1D-CCCs [2-4]. This leads to construction of 2D-QOCCC with elements of smaller order than N^2. Parameter dependencies of 2D-QOCCC on 1D-CCC input code are discussed in this paper later. Benefits of it are also briefly described.

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Paper is organized as follows: Introduction to CCC is given in section II and III for 1D and 2D cases, respectively. In section IV generalized construction is presented. In section V parameter dependence is demonstrated. In section VI advantages of proposed construction are summarized. Conclusion is given in VII.

CCC DEFINITIONS

For the convenience of the reader before defining CCC known definitions of discrete aperiodic cross-correlation function is given [11]:

$$C_{p,r}(s) = \begin{cases} \sum_{u=0}^{T-1-s} a_u^{(p)} a_{u+s}^{(r)}, & 0 \le s \le T-1 \\ \sum_{u=0}^{T-1+s} a_u^{(p)} a_{u+s}^{(r)}, & 1-T \le s < 0 \\ 0, & |l| \ge T \end{cases} \tag{1}$$

where s denotes shift, T denotes period of equally long distinguish sequences ($a_j^{(p)}$) and ($a_j^{(r)}$) of p-th and r-th users consisting of coordinates +1 and -1. $C_{p,r}$ denotes discrete aperiodic autocorrelation function if $p = r$.

Signature is a collection of sequences assigned to one user where autocorrelation function computed through all sequences has ideal property: it is zero for any nonzero shift.

Two signatures are said to be mutually orthogonal if every two complementary sets in the collections are mates of each other.

In CCC sequences of signatures are to be transmitted via independent channels [12]. This allows computing auto- and cross-correlation on receiving end independently for each channel and then to sum up results to obtain overall cross- and autocorrelation. In this way it is possible to achieve ideal cross- and autocorrelation properties.

Because CCCs are formed by more sequences, original definition from [11] has to be slightly modified for CCC:

The *aperiodic autocorrelation* function $\rho_{\mathbf{c}^{(i)}}$ of L long sequence $\mathbf{c}^{(i)}$ is defined as:

$$\rho_{\mathbf{c}^{(i)}}(\tau) = \sum_{l=0}^{L-1} \mathbf{c}^{(i)}(l).\left[\mathbf{c}^{(i)}(l+\tau)\right]^*; \tag{2}$$

where τ denotes the shift.

The *aperiodic cross-correlation* function $\rho_{\mathbf{c}^{(i)},\mathbf{c}^{(j)}}(\tau)$ between two different sequences $\mathbf{c}^{(i)} \in C$ and $\mathbf{c}^{(j)} \in C$ where $i \ne j$:

$$\rho_{\mathbf{c}^{(i)},\mathbf{c}^{(j)}}(\tau) = \sum_{l=0}^{L-1} \mathbf{c}^{(i)}(l).\left[\mathbf{c}^{(j)}(l+\tau)\right]^*; \tag{3}$$

Let's denote *i*-th *signature* in a set *C* of *N* signatures as:

$$\mathbf{c}^{(i)} = (\mathbf{c}_1^{(i)} \quad \mathbf{c}_2^{(i)} \quad \dots \quad \mathbf{c}_E^{(i)}); \quad i = 1,2,\dots N, \tag{4}$$

where each sequence:

$$\mathbf{c}_k^{(i)} = (c_{k,1}^{(i)} \quad c_{k,2}^{(i)} \quad \dots \quad c_{k,L}^{(i)}); \quad k = 1,2,\dots E, \tag{5}$$

is a k-th *element* of it with length L. Each element is a vector in which coordinates are symbols $\left|c_{k,j}^{(i)}\right| = 1$.

The CCC possesses ideal aperiodic auto- and cross-correlation properties. In other words (2) and (3) are equal to zero except for zero shift of aperiodic autocorrelation (2).
Numerous construction methods which lead to CCC with different parameters were recently published, namely the length of elements L alternates e.g. N^2, $2^m N$ or N for maximal number of signatures N using [2], [3] and [4], respectively, where N is power of two. In some applications it is desirable to minimize L [4] keeping N maximal.

2D-CCC

Let $\mathbf{C}$ be a complex matrix of order P made of complex numbers c_{ij} whose absolute values $\left|c_{ij}\right| = 1$:

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1P} \\ \cdots & & & \\ c_{P1} & c_{P2} & \cdots & c_{PP} \end{bmatrix} \tag{6}$$

M_{2D} sets of N_{2D} matrices

$$\left\{\mathbf{C}_1^{(1)}, \mathbf{C}_2^{(1)}, \ldots \mathbf{C}_{N_{2D}}^{(1)}\right\}, \ldots \left\{\mathbf{C}_1^{(M_{2D})}, \mathbf{C}_2^{(M_{2D})}, \ldots \mathbf{C}_{N_{2D}}^{(M_{2D})}\right\} \tag{7}$$

compose a 2D-CCC of order M_{2D} if their autocorrelation and cross-correlation are ideal where functions are defined in Appendix A (18) and (19), respectively. A set of N_{2D} matrices $\left\{\mathbf{C}_1^{(i)}, \mathbf{C}_2^{(i)}, \ldots \mathbf{C}_{N_{2D}}^{(i)}\right\}$ is termed i-th signature of 2D-CCC. A matrix $\mathbf{C}_j^{(i)}$ is termed the j-th element of i-th signature.

CONSTRUCTION OF 2D-QOCCC

Let $\mathbf{C}_{n_{2D}}^{(k_{2D})}$ denote the n_{2D}-th element of the k_{2D}-th signature of the new 2D-QOCCC. It is a matrix with order P. Let $\mathbf{c}_{n_{2D},i}^{(k_{2D})}$ denote its i-th row $i = 1,2,\ldots,P$, $P = L$.
Each element of 2D-QOCCC signature is composed of rows obtained using following equation:

$$\mathbf{c}_{n_{2D},i}^{(k_{2D})} = \mathbf{c}_{n_{\,2\mathrm{D}}}^{[(k_{\,2\mathrm{D}}-1)\bmod M_{1\mathrm{D}}]+1} \times c_{v,i}^{(f)} \tag{8}$$

$$f = \left[(k_{2D} - t) \bmod M_{1D}\right] + 1 \tag{9}$$

$$v = \left[\left(\left\lfloor \frac{k_{2D} - 1}{M_{1D}} + 1 \right\rfloor - 1\right) \bmod N_{1D}\right] + 1 \tag{10}$$

$$t = t + 1 \text{ for } (k_{2D} - 1) \bmod M_{1D}^2 = 0 \tag{11}$$

$$-N_{1D} + 1 \le t \le 0 \tag{12}$$

$$k_{2D} = 1,2,\ldots,M_{1D}^2 N_{1D} \tag{13}$$

$$n_{2D} = 1,2,...,N_{1D} \tag{14}$$

where $\lfloor x \rfloor$ is the greatest integer, which is equal or smaller than x, M_{1D} and N_{1D} denote number of signatures and elements of inputted 1D-CCC, respectively.

Borders and equation are generalized in comparison to original method [1] which uses solely 1D-CCC with element length N^2 [2] for input and generates 2D-QOCCC with element order of N^2. Recently new 1D-CCCs with different lengths were proposed [3-4] which could not be used as input for former method with given algorithm and borders in [1]. This generalized construction accepts any 1D-CCC on input. Furthermore, it is not necessary to input all 1D-CCC signatures. The resulting 2D-QOCCC constructed by inputting a non-optimal 1D-CCC will increase in number of signatures and elements according to conditions described in section IV. Optimal CCC denotes CCC where number of signatures is equal to number of elements of each signature.

PARAMETERS

Given construction generates 2D-QOCCC increased in number of signatures with unmodified number of elements of each signature if compared with 1D-CCC. Number of signatures, elements and element order of 2D-QOCCC generated according to proposed construction depend on parameters of inputted 1D-CCC as follows:

$$M_{2D} = M_{1D}^2 N_{1D} \tag{15}$$

$$N_{2D} = N_{1D} \tag{16}$$

$$L_{2D} = L_{1D} \times L_{1D} \tag{17}$$

where M, N, L are number of signatures, elements and element‘s length for 1D and 2D, respectively.

For recently published 1D-CCC constructions, table 1 shows resulting parameters of 2D-QOCCC.

PARAMETERS Table I

reference	*1D- CCC parameters*			*2D-QOCCC parameters*		
	Signatures	*Elements*	*Length*	*Signatures*	*Elements*	*Order*
2.	M	N	N^2	M^2.N	N	N^2xN^2
3.	M	N	2^mN	M^2.N	N	2^mNx2^mN
4.	M	N	N	M^2.N	N	NxN

where M, N, L are number of signatures, elements and element‘s length for 1D-CCC, respectively.

ADVANTAGES

In [1, 13] a Multicarrier CDMA (MC-CDMA) using 2D-CCC and 2D-QOCCC are proposed in two dimensional time-frequency domain. Considering smaller order of 2D elements *N*-times, 2D channels can be narrowly allocated, thus the available frequency domain can be used effectively for transmission of more signatures.

Each signature in MC-CDMA application can be used for one user, thus the maximal number of concurrently communicating users is given by number of signatures.

Each obtained element of order N in MC-CDMA application has to be transmitted via separate channel [1], e.g. frequency-time zone in 2D introduced in [13] where by unchanged number of channels the 2D-QOCCC needs N-times less channels than 2D-CCC for transmissions of equal number of users. The only one restriction is that QOCCC does not retain fully ideal correlation property. This property is only slightly disregarded which means that the cross-correlation property is non-zero for some possible shifts which can be prevented by forbiding these shifts by used protocol. These non-zero values occur for some vertical shifts within the zero horizontal shift, solely when both signatures have odd or even indexes *(k)*, e.g. 1 and 3 or 2 and 4 etc. Remaining cross-correlation values are zero values, thus ideal.

2D-QOCCC enables achievement of higher spectral efficiency than 2D-CCC with the same order by avoiding unideal cross-correlation vertical shifts. Number of avoided shifts is considerably small if compared with increase of possibly used spectral efficiency.

CONCLUSION

In this paper generalized construction of 2D-QOCCC was presented. This construction can be applied on any recently published 1D-CCC. Generated 2D-QOCCC will increase in number of signatures in comparison to 1D-CCC with retain of unchanged number of elements of each signature according to conditions shown in section IV. Using presented construction and inputting the recently proposed 1D-CCC [3], the 2D-QOCCC with element order N can be generated where achieved order is N-times lower than the original proposed in [1].

APPENDIX A

$$\boldsymbol{\rho}(\mathbf{C},o,p)=\begin{cases}\frac{1}{M.N}\sum_{k=1}^{M-o}\sum_{l=1}^{N-p}\mathbf{c}_{kl}.\mathbf{c}^{*}_{(k+o)(l+p)} & for\, o=0,1,\ldots M-1;\, p=0,1,\ldots N-1\\ \frac{1}{M.N}\sum_{k=1}^{M-o}\sum_{l=1-p}^{N}\mathbf{c}_{kl}.\mathbf{c}^{*}_{(k+o)(l+p)} & for\, o=0,1,\ldots M-1;\, p=-N+1,\ldots -1\\ \frac{1}{M.N}\sum_{k=1-o}^{M}\sum_{l=1}^{N-p}\mathbf{c}_{kl}.\mathbf{c}^{*}_{(k+o)(l+p)} & for\, o=-M+1,\ldots -1;\, p=0,1,\ldots N-1\\ \frac{1}{M.N}\sum_{k=1-o}^{M}\sum_{l=1-p}^{N}\mathbf{c}_{kl}.\mathbf{c}^{*}_{(k+o)(l+p)} & for\, o=-M+1,\ldots -1;\, p=-N+1,\ldots -1\end{cases};\quad (18)$$

$$\boldsymbol{\rho}(\mathbf{C}^{(x)},\mathbf{C}^{(y)},o,p)=\begin{cases}\frac{1}{M.N}\sum_{k=1}^{M-o}\sum_{l=1}^{N-p}\mathbf{c}^{(x)}_{kl}.\mathbf{c}^{(y)*}_{(k+o)(l+p)} & for\, o=0,1,\ldots M-1;\, p=0,1,\ldots N-1\\ \frac{1}{M.N}\sum_{k=1}^{M-o}\sum_{l=1-p}^{N}\mathbf{c}^{(x)}_{kl}.\mathbf{c}^{(y)*}_{(k+o)(l+p)} & for\, o=0,1,\ldots M-1;\, p=-N+1,\ldots -1\\ \frac{1}{M.N}\sum_{k=1-o}^{M}\sum_{l=1}^{N-p}\mathbf{c}^{(x)}_{kl}.\mathbf{c}^{(y)*}_{(k+o)(l+p)} & for\, o=-M+1,\ldots -1;\, p=0,1,\ldots N-1\\ \frac{1}{M.N}\sum_{k=1-o}^{M}\sum_{l=1-p}^{N}\mathbf{c}^{(x)}_{kl}.\mathbf{c}^{(y)*}_{(k+o)(l+p)} & for\, o=-M+1,\ldots -1;\, p=-N+1,\ldots -1\end{cases};\quad (19)$$

where $\mathbf{c}^{*}_{ij}$ is the complex conjugate of $\mathbf{c}_{ij}$.

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