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CONTROL OF OSCILLATIONS IN SECOND-ORDER DIFFERENTIAL EQUATION

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Abstract

The article deals with the control of oscillations in a specific type of second-order differential equations. The purpose of the research is to prove the possibility of oscillation frequency control based on a change in the value of a singular perturbation parameter placed into a mathematical model of a nonlinear dynamical system at the highest derivative. The oscillation frequency change caused by a different value of the parameter is verified by numerically modelling the system.

Key words

oscillation control, second-order differential equation, singular perturbation theory

Introduction

Control of oscillations has recently received a great deal of interest resulting from the associated practical and academic interests. Occurrence of oscillations, encompassed in a wide range of interesting phenomena from the areas of mechanics, mechanical engineering, chemistry, biology, medicine, economics and others makes their control an interesting research question (see e.g. (1, 2, 5) and the references therein).

The problem of accurate prediction and control of oscillations in a wide range of differential equations has not been satisfactorily solved yet, though extensive foundations have been laid, e.g. in (2, 3) and (5).

In (2), the question of oscillation pattern control for the dynamical systems with the multiarmed pitchfork bifurcation was studied and numerically verified, referring to numerous papers dealing with the issue from various approaches, providing a novelty method lying in its straightforward extension to both continuous and discrete nonlinear models with time-delay.

The authors in (3) studied dynamics of the forced singularly perturbed differential equation of Duffing's type, building on the same equation type as herein later. A new method of analysing the nonlinear oscillations based on the dynamic change of coordinates was elaborated.

Similarly, the dynamics of the forced singularly perturbed differential equations of Duffing's type with a potential bounded from above in the presence of a saddle-centre bifurcation was studied in (5). The authors showed that the frequency can be controlled by a small parameter at the highest derivative, as well.

Methods and materials

As stated in (2, 7), there are two main objectives regarding the oscillation control:

(a) To obtain an asymptotically stable zero solution attracting all initial conditions in a suitably large region (regulator problem).

(b) To obtain an asymptotically stable periodic solution with desired properties (such as oscillations at the given amplitudes and frequencies), and attracting all initial states in a suitably large region (oscillator problem).

Based on the research of Vrábeľ et al. (2), we focused our attention on the proof of the existence and the possibility of control of nonlinear oscillations in the dynamical system described as

$$\varepsilon^2 y'' + f(t, y) = 0 \quad [1]$$

$$y(-\delta) = y_0 \qquad y'(-\delta) = y_1 \quad [2]$$

where y_0, y_1 are the initial conditions, $y_\varepsilon(., y_0, y_1)$ is a direct output, t is time, $f(t, y)$ is a function described as [8], ε is a singular perturbation parameter, $0 < \varepsilon \ll 1$ and $\delta > 0$.

A background for modelling the oscillations is provided in the theory of singular perturbations. If the equation [1] is rewritten into a system of three first-order equations, the following system is obtained

$$\varepsilon y' = w$$

$$\varepsilon w' = -f(t, y) \quad [3]$$

$$t' = 1$$

$$y(-\delta) = y_0 \qquad w(-\delta) = \varepsilon y_1 \quad [4]$$

Applying the limit$\varepsilon \rightarrow 0^+$, the system [3] is reduced to an algebraic-differential reduced system of the form

$$0 = w$$

$$0 = -f(t, y) \quad [5]$$

$$t' = 1$$

In the next step, the substitution $\tau = \frac{t}{\varepsilon}$ is introduced, transforming the system [3] to the system

$$\frac{\partial y}{\partial \tau} = w$$

$$\frac{\partial w}{\partial \tau} = -f(t, y) \qquad [6]$$

$$\frac{\partial t}{\partial \tau} = \varepsilon$$

Applying the limit $\varepsilon \to 0^+$, the system [6] is reduced to the associated system with t playing the role of a parameter

$$\frac{\partial y}{\partial \tau} = w$$

$$\frac{\partial y}{\partial \tau} = -f(t, y) \qquad [7]$$

$$\frac{\partial t}{\partial \tau} = 0$$

As, regarding the level of phase space structure, both systems agree when $\varepsilon \neq 0$, but differ significantly in the limit when $\varepsilon = 0$, the methodical base for applying the singular perturbation theory is provided, since the main goal of this theory is to understand the structure in the full system when $\varepsilon \neq 0$ (4).

Results

As an example, let us consider the oscillations in the dynamical system described as [1] and [2], where $f(t, y)$ has the form:

$$f(t, y) = \begin{cases} y^3 & \text{for } t \in \langle -\delta, 0 \rangle \\ \big(y - (e^t - 1)\big) y \big(y + (e^t - 1)\big) & \text{for } t \in \langle 0, \infty \rangle \end{cases} \qquad [8]$$

ε is a singular perturbation parameter, $\varepsilon \to 0^+, t$ is time and $\delta > 0$. The symmetric manifold representing $f(t, y) = 0$ is depicted in Figure 1.

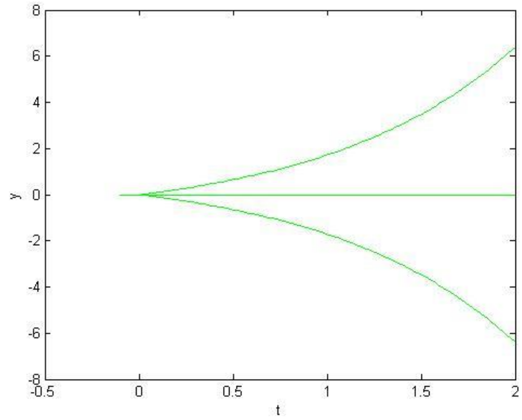


Fig. 1 *Symmetric manifold* $f(t,y) = 0$

Under varying the singular perturbation parameter ε, the graph of the studied equation differs significantly. It is necessary to point out, that these oscillations are very sensitive to the value of the singular perturbation parameterε, as it comes out from the following figures. When comparing Figure 2 with Figure 3, the difference in the parameter ε is on the fifth decimal place; hence the oscillations are extremely different. Secondly, the analysis of Figure 4 shows that the frequency of the oscillations falls with the rising value of the singular perturbation parameter ε.

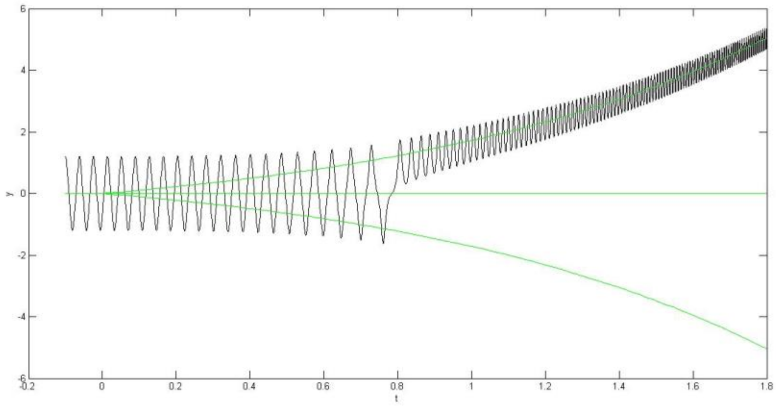


Fig. 2 *Numerical solution of* $\varepsilon^2 y'' + f(t,y) = 0$, $y(-\delta) = y_0$, $y'(-\delta) = y_1$, $y_0 = 1.2$, $y_1 = 0$, $\delta = 0.1$, $T = 1.8$, $\varepsilon = 0.00612$

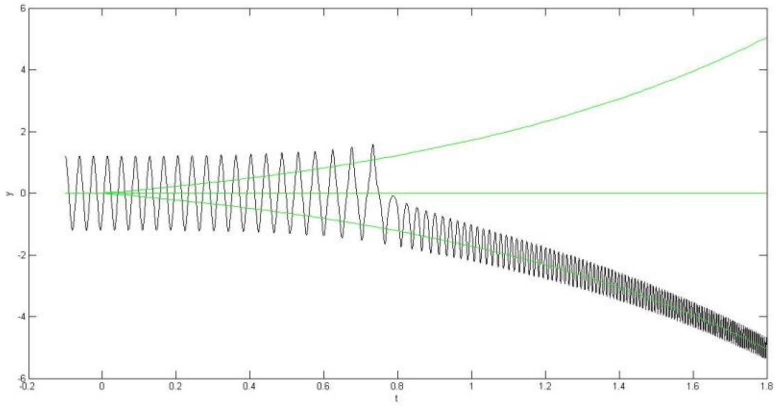
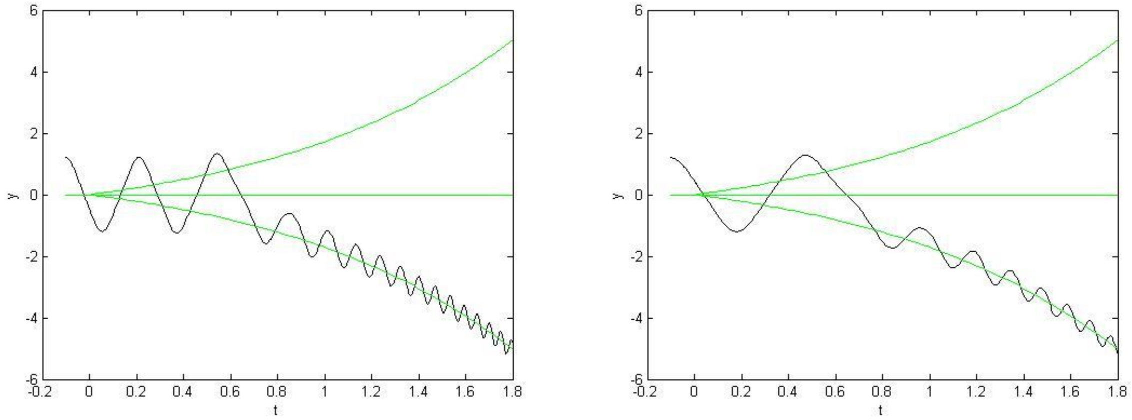


Fig. 3 *Numerical solution of* $\varepsilon^2 y'' + f(t,y) = 0$, $y(-\delta) = y_0$, $y'(-\delta) = y_1$, $y_0 = 1.2$, $y_1 = 0$, $\delta = 0.1$, $T = 1.8$, $\varepsilon = 0.00613$



(a) (b)

Fig. 4 *Numerical solution of* $\varepsilon^2 y'' + f(t,y) = 0$, $y(-\delta) = y_0$, $y'(-\delta) = y_1$, $y_0 = 1.2$, $y_1 = 0$, $\delta = 0.1$, $T = 1.8$, $\varepsilon = 0.05$ *(a)*, $\varepsilon = 0.09$ *(b)*

The data for figures has been worked out by using MATLAB computer system (6). The source code is available from the authors upon request.

Discussion

The numerical results confirm that different oscillation frequency can be achieved as a result of the change in the singular perturbation parameter ε at the highest derivative. When comparing Figure 2 with Figure 3, it is obvious, that the oscillations differ significantly, although there is only a minor change in the parameter ε . It is as well obvious, that the frequency of the oscillations is inversely proportional to the value of the singular perturbation parameter ε.

Conclusion

We demonstrated the possibility of oscillations control based on a small fitting parameter placed into a mathematical model of a nonlinear dynamical system. The results were modelled using MATLAB software. However, in order to obtain the oscillations with specific properties, it is necessary not only to deeply understand each individual system, but also to identify and meet particular system-behaviour requirements. The future research will be therefore oriented on altering properly the initial conditions as well as the value of the singular perturbation parameter.

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