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AN EVALUATION OF MATERIAL PROPERTIES USING EMA AND FEM

Rastislav ĎURIŠ, Eva LABAŠOVÁ

SLOVAK UNIVERSITY OF TECHNOLOGY IN BRATISLAVA,
FACULTY OF MATERIALS SCIENCE AND TECHNOLOGY IN TRNAVA,
INSTITUTE OF APPLIED INFORMATICS, AUTOMATION AND MECHATRONICS,
ULICA JÁNA BOTTU 2781/25, 917 24 TRNAVA, SLOVAK REPUBLIC
e-mail: rastislav.duris@stuba.sk, eva.labasova@stuba.sk

Abstract

The main goal of the paper is the determination of material properties from experimentally measured natural frequencies. A combination of two approaches to structural dynamics testing was applied: the experimental measurements of natural frequencies were performed by Experimental Modal Analysis (EMA) and the numerical simulations, were carried out by Finite Element Analysis (FEA). The optimization methods were used to determine the values of density and elasticity modulus of a specimen based on the experimental results.

Key words

experimental modal analysis, finite element method, material properties, optimization procedures, ANSYS

INTRODUCTION

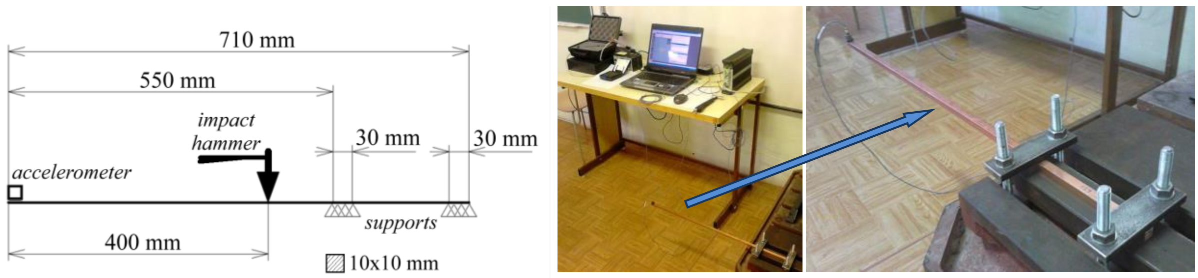
The basic purpose of modal identification is to determine modal parameters from experimental data. The general method is to use input-output modal identification where the modal parameters are found by fitting a linear model to the Frequency Response Function (FRF), a function relating the excitation frequency spectrum $F(f)$ to the response frequency spectrum $X(f)$

$$X(f) = H(f)\,F(f)\,.$$

The paper deals with numerical determination of material properties (elastic modulus, density) using experimentally measured modal parameters and optimization techniques implemented in the finite analysis software (1). The experimental measurement of modal properties of structures, including the data acquisition, Fourier transform and subsequent data analysis, is referred to as Modal Analysis. This process characterizes the dynamic behavior of mechanical structures in regards to modal properties. Mobility measurement (accelerance) is often performed using impact hammer excitation. It is a very fast method to perform the transient test for determining the dynamic properties of the structure. Advantages of impact testing are that it usually requires a low number of averages; no fixtures are required; it is more flexible when compared with the shaker exciter and finally it is easy to use. Disadvantages of using the impact hammer are: the input force signal creates high crest factors and therefore the impact technique is inapplicable to nonlinear systems or systems sensitive to stress peaks, it requires special windowing (weighting) functions, it can give deceptive results, it is easy to misinterpret the signals and it has limited control of the excitation bandwidth (3, 4, 7).

EXPERIMENTAL MEASUREMENTS

As the testing structure, three dimensionally identical beams of different materials were used. The applied material of beams were: steel, copper and brass. Fig. 1a shows the model and geometric dimensions of the cantilever beam with a square cross-section used for experimental modal analysis. The excitation was conducted by impact hammer Brüel&Kjær Type 8206-001 with an aluminium tip. Transducer Type 4508-B mounted in assembly clips UA-1407 were used as the acceleration sensor. In Fig.1b is shown the measuring system, the location of the sensor and the method of fixation of the beam. Experiments were carried out using Brüel&Kjær PULSE software.



a) b)

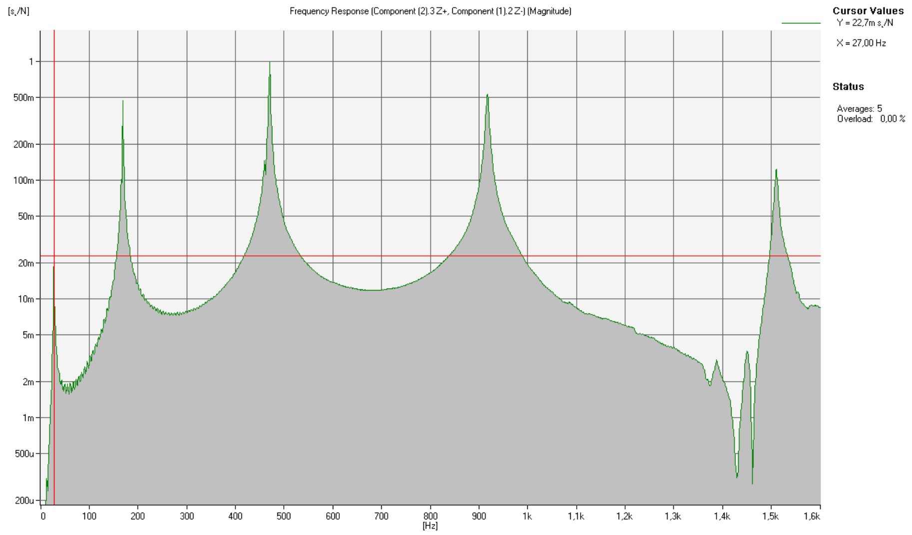
Fig. 1 *The measuring system: a) the geometrical parameters of test beams, location of accelerometer and position of impact point; b) experiment layout with PULSE 3560-B-120*

The dynamic behavior of the structure in a given frequency range can be modelled as a set of individual modes of vibration. The structure is assumed to behave as a time-invariant linear system. The parameters that describe each mode are: natural frequency, modal damping and mode shape. The modal parameters can be extracted from the set of Frequency Response Function (FRF) measurements between one or more reference positions and between multiple measurements positions required in the model (2).

To determine the resonance frequencies of the beam, a single accelerometer is mounted at the free end of the cantilever beam. For this classic case of a single input, FRF H_{ij} gives dependence of the output at any DOF i (vibration response – X), on the input at DOF j (force excitation – F) $H_{ij} \equiv \frac{X_i}{F_j}$. The FRF H_{ij} can be estimated using the classical response function such as

$$H_1(f) \equiv \frac{G_{FX}(f)}{G_{FF}(f)} \qquad \text{or} \qquad H_2(f) \equiv \frac{G_{XX}(f)}{G_{XF}(f)},$$

where $G_{FX}(f)$ is the one-sided cross-spectrum between force and response, $G_{XF}(f)$ is the one-sided cross-spectrum complex conjugate of $G_{FX}(f)$, $G_{FF}(f)$ and $G_{XX}(f)$ are the one-sided autospectra of excitation signal and output, respectively. The complex estimation function $H_1(f)$ has the ability, to eliminate the influence of uncorrelated noise at the output due to averaging, whereas $H_2(f)$ has the ability, to eliminate the influence of uncorrelated noise at the input (4). To minimize the effect of noise in the response function $H_1(f)$ was used.



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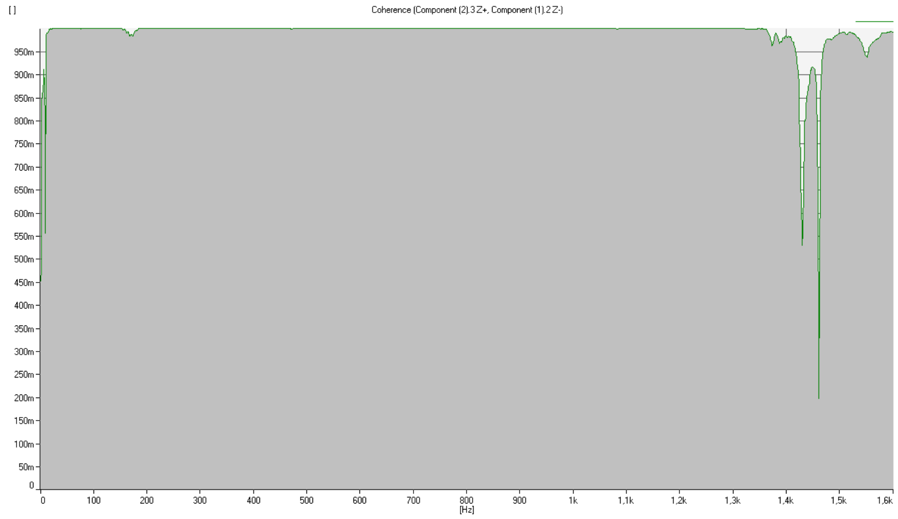

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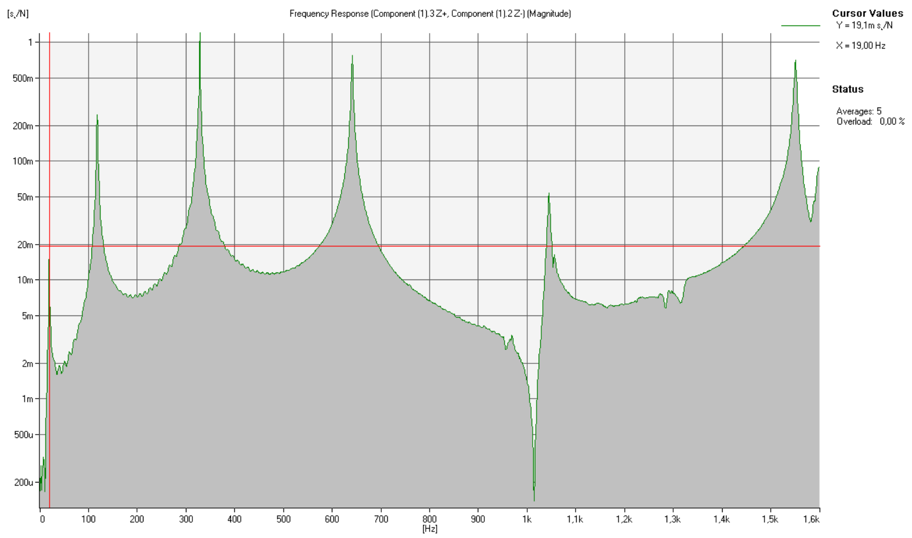
Fig. 2 *Estimation of (a) frequency response function $H_1(f)$ and (b) coherence functions for measurement on steel beam*



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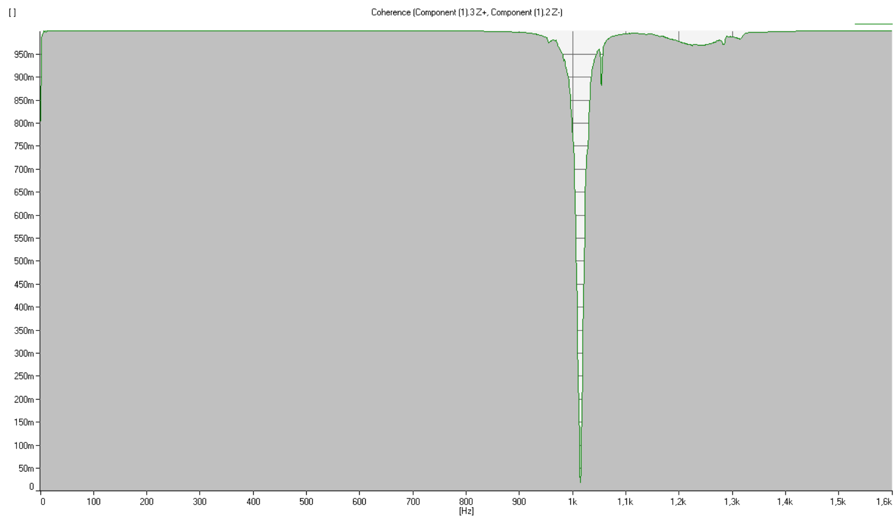

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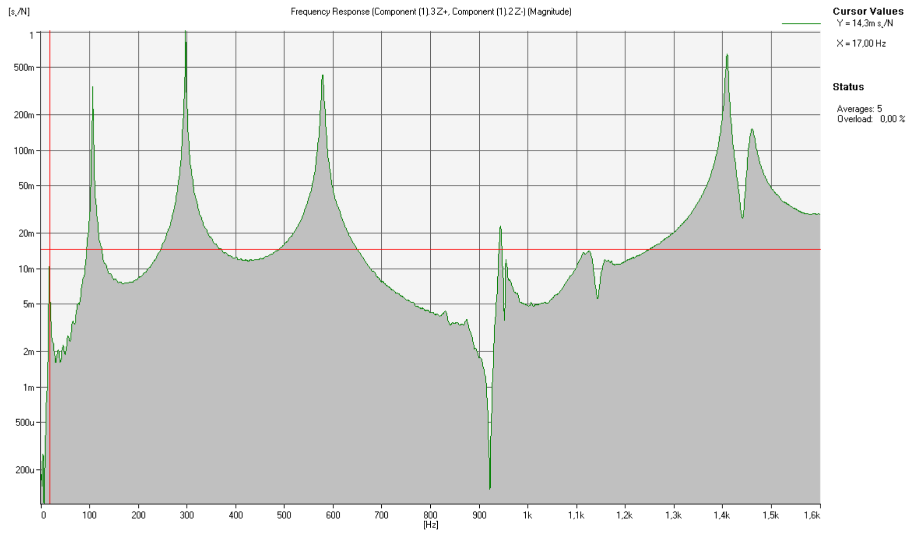
Fig. 3 *Estimation of (a) frequency response function $H_1(f)$ and (b) coherence functions for measurement on copper beam*



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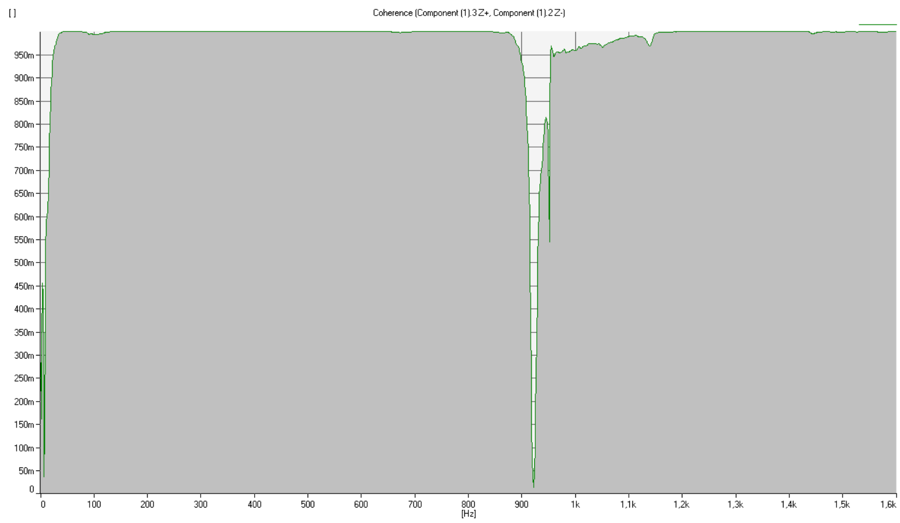


b)

Fig. 4 *Estimation of frequency response functions $H_1(f)$ and coherence functions for measurement on brass beam*

In Figs. 2a, 3a and 4a the estimate of FRF $H_1(f)$ from measurement with a single accelerometer for different beam materials can be seen. Part b) of the figure plots of coherence functions confirm good linearity and noiselessness of measurements in the considered frequency range (limited by ≈1 kHz) for all experiments. The identified modal frequencies are summarized in the following table.

Table 1 The natural frequencies of specimens obtained by experimental modal analysis

beam with 1 attached accelerometer	**1st natural frequency**	**2nd natural frequency**	**3rd natural frequency**	**4th natural frequency**
steel	27 Hz	168 Hz	469 Hz	916 Hz
copper 99,85%	19 Hz	117 Hz	328 Hz	640 Hz
brass CuZn37Pb2	17 Hz	105 Hz	296 Hz	578 Hz

DETERMINATION OF MATERIAL PROPERTIES USING FEM AND OPTIMIZATION PROCEDURES

If we consider the undamped natural vibration, the equation of eigenvalue problem has the form

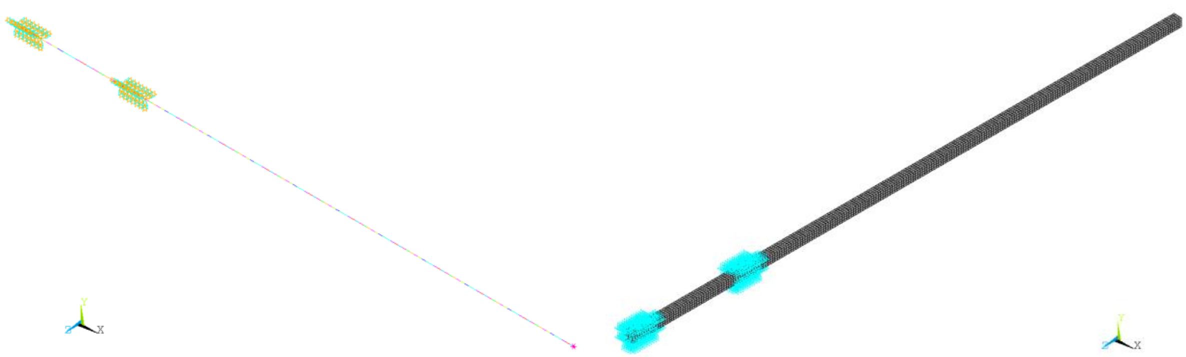
$$(\mathbf{K} - \mathbf{\Omega}^2\mathbf{M})\,\mathbf{\Phi} = \mathbf{0}\,,$$

where $\mathbf{K}$ is the stiffness matrix of the structure, $\mathbf{M}$ is the mass matrix, $\mathbf{\Omega}^2$ is a diagonal matrix of squares of natural circular frequencies and $\mathbf{\Phi}$ is the matrix of eigenvectors (mode shapes). This equation was solved using the finite element software ANSYS with application of design optimizing procedure.

For the numerical simulations, two FE models in ANSYS were created:

- 1D beam model meshed into 140 elements (BEAM188 configured as 2-node isoparametric element with constant cross-sectional area) – Fig. 5a,
- 3D model using fine mesh of 2944 SOLID185 elements – Fig. 5b.

In FE models, the mass of transducer and mounting clip were taken into account due to their significant influence on modal frequencies.



a) b)

***Fig. 5** Finite element models with applied boundary conditions*

Optimization methods are techniques that seek minimization of the objective function subject to constraints. The following optimization methods are available in the ANSYS software: the sub-problem approximation method, the first order method and an external user-supplied method.

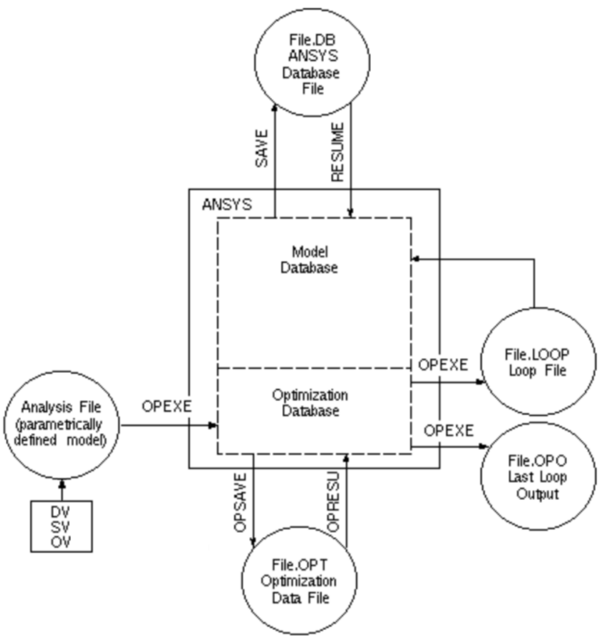


Fig. 6 *ANSYS optimization procedure [ANSYS Help]*

ANSYS categorizes three types of variables for design optimization: *design variables* - independent variables that directly affect the design objective, *state variables* - dependent variables necessary to constrain the design that change as a result of changing the DVs and *objective variable* - single variable in the optimization that needs to be minimized.

In the design optimization procedure (Fig. 6) experimental values of four natural frequencies were used as independent design variables. As the state variables material parameters (density, elastic modulus) were chosen, the objective variable was defined in the form of a simple frequency error (*FER*) function (5, 8)

$$FER = \sum_{i=1}^{n} [(f_{i,ANSYS}) - (f_{i,MEASURED})]^2 ,$$

where n is the number of considered natural frequencies.

Using the first order optimization method (which uses derivatives of the functions to achieve the optimum) implemented in the ANSYS software, the input density and elastic modulus in matrices **M** and **K** were determined to obtain modal frequencies comparable with experimental results (6). The performing of numerical optimization procedures resulted in the values of material properties in Table 2. Calculated results of material parameters are compared with the results of experimental measurements for density. Reference values of elastic moduli of used materials are taken from literature.

Table 2 Material properties calculated by numerical analyses of the cantilever beam compared to values obtained by experimental measurement and from literature

material		**values calculated by FEM**		**measured or from the literature taken values**	
	element	ρ [kg.m^{-3}]	E [GPa]	ρ [kg.m^{-3}]	E [GPa]
steel	BEAM188 SOLID185	7575 7636	180 178	7595	190-210
copper	BEAM188 SOLID185	8960 8979	117 115	8945	117
brass	BEAM188 SOLID185	8340 8395	93 91	8391	99

In Table 3 are the compared measured and numerically computed modal frequencies after substitution of “optimized” values of density and elastic modulus into FEM models. The good agreement of experimentally obtained results and FEM computed eigenfrequencies with optimized mass-elastic material parameters can be seen.

Table 3 Comparison of natural frequencies obtained by experimental measurement against to results of numerical analyses of the cantilever beam with optimized material properties

steel	**experiment**	**FEM – BEAM188** **FEM – SOLID185**	**difference**
1st natural frequency	27 Hz	26,63 Hz 26,54 Hz	1,36 % 1,70 %
2nd natural frequency	168 Hz	166,84 Hz 166,53 Hz	0,69 % 0,88 %
3rd natural frequency	469 Hz	466,48 Hz 466,19 Hz	0,54 % 0,60 %
4th natural frequency	916 Hz	911,85 Hz 912,16 Hz	0,42 % 0,45 %

copper	**experiment**	**FEM – BEAM188** **FEM – SOLID185**	**difference**
1st natural frequency	19 Hz	18,71 Hz 18,63 Hz	1,55 % 1,94 %
2nd natural frequency	117 Hz	117,18 Hz 117,90 Hz	0,15 % 0,77 %
3rd natural frequency	328 Hz	327,72 Hz 327,29 Hz	0,09 % 0,22 %
4th natural frequency	640 Hz	640,84 Hz 640,59 Hz	0,13 % 0,09 %

brass	**experiment**	**FEM – BEAM188** **FEM – SOLID185**	**difference**
1st natural frequency	17 Hz	17,38 Hz 17,22 Hz	2,23 % 1,29 %
2nd natural frequency	105 Hz	108,88 Hz 108,77 Hz	3,70 % 3,59 %
3rd natural frequency	296 Hz	304,51 Hz 302,60 Hz	2,88 % 2,23 %
4th natural frequency	578 Hz	595,46 Hz 592,28 Hz	3,02 % 2,47 %

DISCUSSION

By comparing the material properties calculated from the eigenfrequencies to reference values one can see that the values differ only in the range about 6% for the elastic moduli and less than 1 % for the density. The difference between numerically computed modal frequencies after back substitution of “optimized” material properties into FEM models are less than 3,7 % for each of the materials considered. When using several acceleration sensors, the MIMO method of FRF estimation can be used to refine the results. Then as the objective function modal assurance criterion (*MAC*) relating the degree of consistency between calculated and reference (measured) modal shapes can be used.

CONCLUSION

In the study, the system identification approach for measuring the material properties of homogeneous elastic materials from beam flexural vibration experimental measurements has been presented. The eigenfrequencies of the in-plane loaded cantilever beam were measured by EMA. Based on these results selected material properties of specimens were determined using design optimization in ANSYS code. The results were compared against values given by experimental measurement of those available in the literature. The presented method allows for further information about frequency dependency of the material properties. It can be concluded that the proposed procedure is effective and usable to identify the basic mechanical properties of materials.

Acknowledgement

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ORCID:

Rastislav Ďuriš 0000-0002-5984-8730
Eva Labašová 0000-0001-9055-1233