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INVERTING MATRIX USING A GRAPH IN THE THEORY OF DYNAMIC SYSTEMS

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Abstract

To obtain the transfer equation in dynamic system theory, we need to calculate an inverse matrix. In this work we show both the classic methods and the methods used in automation theory of inverse matrix calculation. However, they are both time consuming and difficult. In the second section, we will show a simpler method. We will define how to create a graph for a given matrix and how to invert it. The matrix for the inverse graph is inverse of the original. We illustrate this method on an example and compare it to a classic solution.

Key words

dynamic system, inverse matrix, graph, inverting using graph

INTRODUCTION

In dynamic system theory, we often use a mathematical description of these systems. To find the transfer equation of a dynamic system, we first need to calculate an inverse matrix. Computing the inverse matrix by hand can be (in case of larger matrices) tedious. In the first section, we will show the most famous computation methods from linear algebra and automation theory. The main part of this paper is the second section, in which we will show how to calculate an inverse matrix by inverting a graph.

INVERSE MATRIX IN THE THEORY OF DYNAMIC SYSTEM

The internal description of a dynamic system is the relation between all of the variables of the system and is defined using state equations. The external description of the dynamic system is the relation between the input and output variables. In the case of a stationary linear system it is represented by a transfer matrix.

The state equations of a continuous linear system with the starting condition $x(0)=0$ are

$$\dot{x}(t)=Ax(t)+Bu(t)$$
$$y(t)=Cx(t)+Du(t).$$

The transfer matrix of a system can be calculated using the relation $G(s)=C\cdot(sI-A)^{-1}\cdot B+D$. The most "cumbersome" part in this formula, at least as time consumption is concerned, is the calculation of the inverse matrix.

While solving a state equation we need to calculate an inverse matrix to the matrix $(s\cdot I-A)$. We can use any of the procedures known from the basic course of mathematics. We can calculate it using two procedures:

a) We adjust the matrix $(A|I)$ using either column or row elementary operations to get a resulting matrix $(I|A)$.

b) Using the formula

$$A^{-1}=\frac{1}{|A|}\cdot\begin{pmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \vdots & \vdots & & \vdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{pmatrix}.$$

However, they are, especially for larger matrices, rather lengthy.

It is much more efficient to use the process shown in the paper (8). We will show it in a way that is understandable for a reader who doesn´t possess deeper knowledge of linear algebra.

We will first calculate the characteristic polynomial of matrix A. We will get it by calculating the determinant of matrix $(s\cdot I-A)$. After adjusting it to a normalized form we end up with a polynomial

$$s^n+a_{n-1}s^{n-1}+a_{n-2}s^{n-2}+\cdots+a_{n-1}s+a_{0.}.$$

It´s coefficients are $1,a_{n-1},\cdots,a_0$. Auxiliary matrices (we can denote them e.g. $R_0,R_1,\cdots,R_{n-1}$ can be calculated using the recurrent relations:

$$R_0=I$$
$$R_1=A\cdot R_0+a_{n-1}\cdot I$$
$$\vdots$$
$$R_{n-1}=A\cdot R_{n-1}+a_1\cdot I.$$

Then we calculate the inverse matrix

$$(sI-A)^{-1}=\frac{1}{|sI-A|}\left(R_0s^{n-1}+R_1s^{n-2}+\cdots+R_{n-1}\right).$$

The comparison of these methods on selected examples with solutions in matlab can be found in (3). The problem of finding inverse matrices is also addressed in papers (2), (6), (7). In the next section, we will describe (and show on an example) a simple method for inverse matrix calculation using graph theory.

INVERTING USING GRAPHS

Some types of special regular matrices can be inverted using graphs.

We can assign every matrix A a graph $G=(V,H)$ in such a way that we consider matrix A a matrix of adjacencies of graph G. The inverse to every (rated in general) graph G with n vertexes with rating 1 we define the matrix $A=(a_{ij})$ of graph G as a square matrix of the n degree, where

$$a_{ij}=\begin{cases} l(v_i,v_j) \text{ if } (v_i,v_j)\in(G) \\ \text{else } 0 \end{cases}.$$

We will say that a graph contains a 1 – factor, if a factor of graph G exists, such that every vertex has the degree 1. With the symbol $M(G)$ we will denote 1 – factor of graph G (in such a case where it is contained in G).

Graph H is called an inverse to graph G, if the adjacency matrix $A(H)$ of graph H has it´s eigenvalues inverse to the eigenvalues of matrix $A(G)$. We will call a graph invertible if it has at least one inverse graph (5).

From now on, we will only talk about matrices that are an adjacency matrix of an invertible graph.

We can say that path P is alternating in G with regards to $M(G)$, if it fulfils at least one of the following criteria:

i) P is length 1 (it is an edge)

ii) From every pair of neighboring edges in P one of them is in $M(G)$ and P begins and ends with an edge that is not in $M(G)$.

We will say that P

is a pair (odd) alternating path, if P contains a pair (odd) number of edges not contained in $M(G)$.

Let G be a bi-graph (bi-partial graph) without multiplicative edges with only one 1 – factor $M(G)$. Let u,v be the vertexes in graph G, then $p_+(u,v)$ ($p_-(u,v)$respectively) denotes the number of all pair (odd) alternating paths linking vertices u and v.

Let A be a matrix of adjacencies of an invertible graph. We will obtain an inverse matrix by [5]:

To matrix A we will assign graph G such that A is the adjacency matrix of G.

For graph G we will create graph G^{-1} with the degree l where $V(G)=V(G^{-1})$ and the edge $(u,v)\in H(G)$ if and only if $p_+(u,v)\neq p_-(u,v)$ and $l(u,v)=p_+(u,v)-p_-(u,v)$.

Example:

We will specify an inverse matrix to matrix A using a graph construct. We will consider this matrix a bi-partial matrix of graph G as shown in fig. 1. The relation between a bi-partial matrix and a transfer matrix can be found in (4).

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

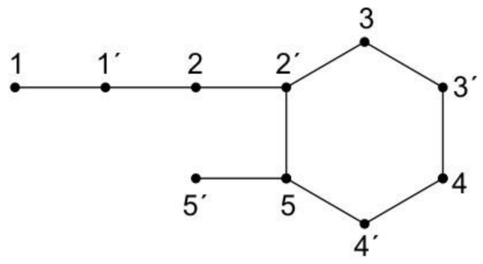

Fig. 1 *Representation of* G

Using the construct we will create a rated inverse graph to graph G, the finished graph is shown in fig. 2. We will rewrite the ratings into the matrix, and the resulting matrix is inverse to matrix A.

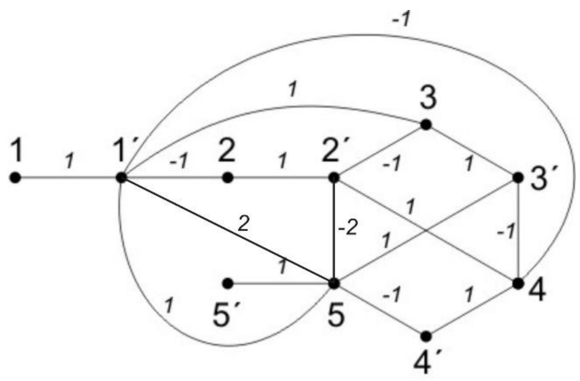

Fig. 2 *Representation of* G^{-1}

$$A^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ -1 & 1 & -1 & 1 & 0 \\ 2 & -2 & 1 & -1 & 1 \end{pmatrix}$$

For the sake of comparison, we will solve this problem using a classic approach. Due to the fact that the matrix is almost diagonal, the simplest method will be by using the properties of equality.

$$(A|I)=\left(\begin{array}{ccccc|ccccc}1&0&0&0&0&1&0&0&0&0\\1&1&0&0&0&0&1&0&0&0\\0&1&1&0&0&0&0&1&0&0\\0&0&1&1&0&0&0&0&1&0\\0&1&0&1&1&0&0&0&0&1\end{array}\right)\sim\left(\begin{array}{ccccc|ccccc}1&0&0&0&0&1&0&0&0&0\\0&1&0&0&0&-1&1&0&0&0\\0&1&1&0&0&0&0&1&0&0\\0&0&1&1&0&0&0&0&1&0\\0&1&0&1&1&0&0&0&0&1\end{array}\right)\sim$$

$$\sim\left(\begin{array}{ccccc|ccccc}1&0&0&0&0&1&0&0&0&0\\0&1&0&0&0&-1&1&0&0&0\\0&0&1&0&0&1&-1&1&0&0\\0&0&1&1&0&0&0&0&1&0\\0&0&0&1&1&1&-1&0&0&1\end{array}\right)\sim$$

$$\sim\left(\begin{array}{ccccc|ccccc}1&0&0&0&0&1&0&0&0&0\\0&1&0&0&0&-1&1&0&0&0\\0&0&1&0&0&1&-1&1&0&0\\0&0&0&1&0&-1&1&-1&1&0\\0&0&0&1&1&1&-1&0&0&1\end{array}\right)\sim$$

$$\sim\left(\begin{array}{ccccc|ccccc}1&0&0&0&0&1&0&0&0&0\\0&1&0&0&0&-1&1&0&0&0\\0&0&1&0&0&1&-1&1&0&0\\0&0&0&1&0&-1&1&-1&1&0\\0&0&0&0&1&2&-2&1&-1&1\end{array}\right)=(I|A^{-1})$$

CONCLUSION

Inverting matrices via the use of graphs is a simple and concise method for calculating inverse matrices. We have shown this method for matrices consisting of ones and zeros. By using other values, we can extend this method for matrices consisting of whole numbers. This result can be found in paper (9).

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