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KINEMATICS SIMULATION OF THE CARDAN SHAFT FOR INVESTIGATION OF THE CARDAN ERROR IN CATIA V5

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Abstract

The goal of this paper is the creation of kinematic systems of the cardan shaft in the CAD/CAM/CAE system CATIA V5 and analysis of three cases of assembly to determine upon which, angular accelerations had been observed between the input driving shaft, central cardan shaft and output driven shaft. The scientific result of this paper was to confirm the presence of cardan error and how this type of error can be successfully eliminated.

Key words

kinematic system, kinematic pair, cardan shaft, cardan joint, CATIA V5

INTRODUCTION

The kinematic mechanism is defined as a set of parts which are mutually movably connected together to perform the desired movement. Their main function is to transmit or change motion (displacement to rotation and vice-versa) and to transmit the power. By changing the speed or the type of movement is changed usually also size of the transmitted power (1). The one of all mechanisms type is also articulated joint mechanism. Into this category of mechanism belongs for example cardan shaft. The cardan shaft is the shaft which connects together two shafts which are not placed coaxial or their relative position can be changed during process time. The articulated joint mechanisms are used almost in the automotive industry (7).

The first known application of the universal joint occurred in China more than 2,000 years ago. The Chinese had invented what we call “gimbals,” a series of interlocking rings within a device that allowed a candle placed in the center to remain upright regardless of the device’s position. Today, gimbals are used to keep ships’ compasses level and as components in gyroscopes. In 1545, Italian mathematician Girolamo Cardano theorized that the principle of gimbals could be used to transmit rotary motion through an angle. This theory was developed over many years into today's Cardan joint (also known as Hook’s joint).

THEORETICAL BASE

By using one universal joint (Fig. 1), the output leads to the uneven distribution of the movement known as the cardan error. During one turn of the driving shaft occurs two times acceleration and two times deceleration on the driven shaft. If axial deviation exists between the input and output, then the driven shaft rotates at an uneven rotational speed ω_1' while the driving shaft has a constant rotational speed ω_1. The acceleration and deceleration of the driven shaft is moved continuously along the sinusoidal curve shape (Fig. 1.) (2, 8, 9).

Therefore, use of the kinematic mechanism with one universal joint is possible only for small axis deviation α. If the axis deviation α will increase, then the value of angular velocity and angular acceleration of the driven shaft will also increase during the rotation of the driven shaft. This can lead to the formation of undesirable vibrations that are transferred to the environment (3, 4).

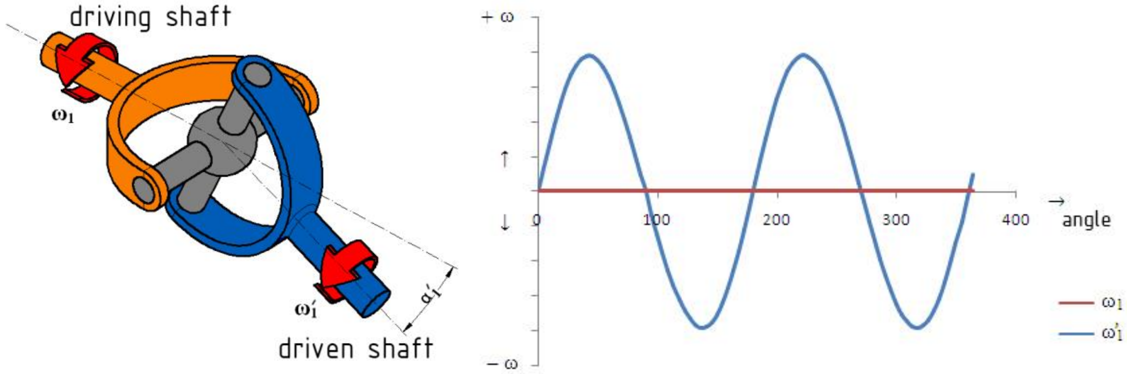

Fig. 1 *Simple model of universal joint and graph of the angular velocity for both shafts*

The kinematics of the universal joint is described through a spherical triangle which consists of a path of the points on the axes of the universal joint.

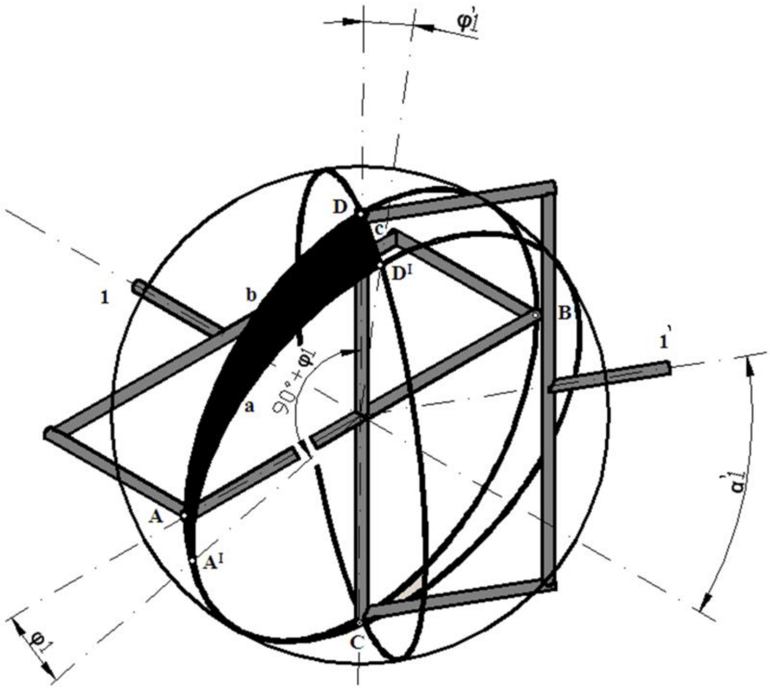

Fig. 2 *Spherical triangle of the universal joint*

In the case of the spherical triangle (Fig. 2), cosine theorem is applied

$$\cos a = \cos b . \cos c + \sin b . \sin c . \cos\alpha_1 \qquad [1]$$

If $a = 90°$, $b = 90° + \varphi_1$, $c = \varphi_1'$ then

$$\frac{\sin\varphi_1 . \cos\varphi_1'}{\cos\varphi_1 . \cos\varphi_1'} = \frac{\cos\varphi_1 . \sin\varphi_1' . \cos\alpha_1}{\cos\varphi_1 . \cos\varphi_1'}$$

$$\mathrm{tg}\varphi_1 = \mathrm{tg}\varphi_1' . \cos\alpha_1 \qquad [2]$$

From the equations, it is clearly seen that if the angle α_1 is bigger, subsequently the angle φ_1 will be smaller and the difference between angles φ_1' and φ_1 will be bigger. The equation for the solution of the angular velocity has been obtained through derivation of the equation [2].

$$\omega_1' = \frac{\cos\alpha_1}{1 - \sin^2\alpha_1 . \cos^2\varphi_1} \cdot t \qquad [3]$$

Cardan error can be eliminated by using three shafts which are coupled together with two universal joints.

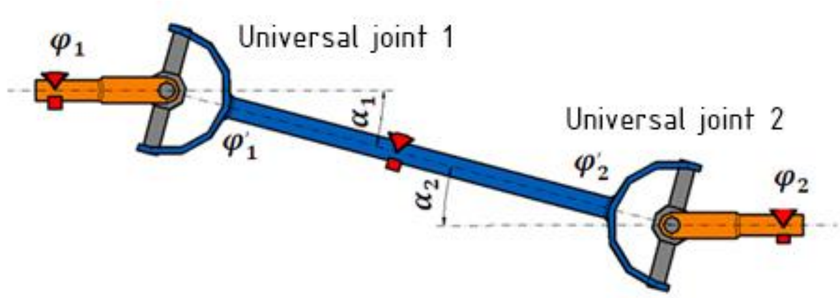

Fig. 3 *Cardan shaft with two universal joints*

From Fig. 3 and the equation [2] are obtained these equations

$$\mathrm{tg}\varphi_1 = \mathrm{tg}\varphi_1' . \cos\alpha_1 \qquad [4]$$

$$\mathrm{tg}\varphi_2' = \mathrm{tg}\varphi_2 . \cos\alpha_2 \qquad [5]$$

Because the angle between the driving shaft fork of the first universal and driving shaft fork of the second universal joint equals 90° it is possible to rewrite the equation [5] into the shape

$$\mathrm{tg}(90 + \varphi_1') = \mathrm{tg}(90 + \varphi_2) . \cos\alpha_2$$

From the equation after adjustment is obtained

$$\mathrm{tg}\varphi_1' = \mathrm{tg}\varphi_2 . \frac{1}{\cos\alpha_2} \qquad [6]$$

Inserting equation [6] into equation [4]

$$\mathrm{tg}\varphi_1 = \mathrm{tg}\varphi_2 . \frac{1}{\cos\alpha_2} . \cos\alpha_1 \qquad [7]$$

If $\alpha_1 = \alpha_2$, then

$$\mathrm{tg}\varphi_1 = \mathrm{tg}\varphi_2 \tag{8}$$

This means that the rotation of the driving shaft φ_1 equals the rotation of the driven shaft φ_2 .

KINEMATICS SIMULATION OF THE CARDAN SHAFT IN CATIA V5

The construction of the universal joints allows transferring torque moment between two rotating abaxial shafts. In some cases, this allows axle shift (2). The most commonly used joint is the universal joint shown in Fig. 4. In the motor vehicles are used universal joints for maximal axes deviation 8°. Special design allows also greater deviation of the axes (5, 6).

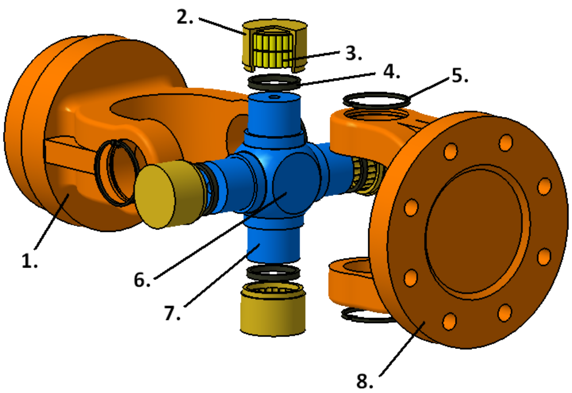

Fig. 4 *3D model of the universal joint (1 – driving fork, 2 – bearing shell, 3 – needle bearing, 4 – sealing, 5 – lock ring, 6 – cross, 7 – joint bolt, 8 – driven fork)*

Kinematic simulation was made using the CAD/CAM/CAE system CATIA V5 on the model of the cardan shaft which contains two universal joints (Fig. 5). Between all connections with bearings, the revolute type of joint and one prismatic type of connection for central cardan shaft was used, which allows adjustment of the relative position between two parts of the cardan shaft through castellated shaft connection. All degrees of freedom were blocked for the frame.

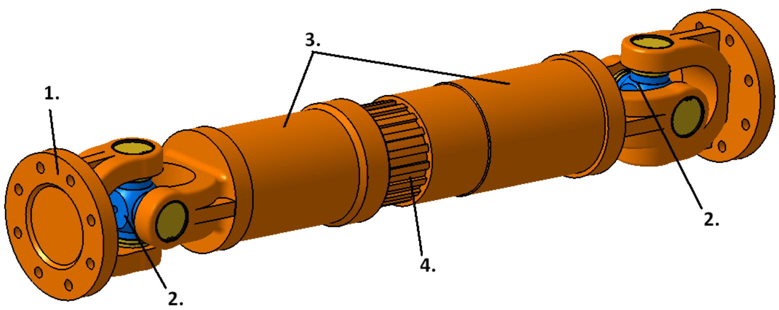

Fig. 5 *3D model of the commonly used cardan shaft (1- flange, 2 – universal joint, 3 – cardan shaft, 4- castellated shaft)*

In the first case, kinematic analysis of the cardan shaft sloped at an angle 15°, was simulated. The input and output shafts were parallel during analysis as is shown in Fig. 6.

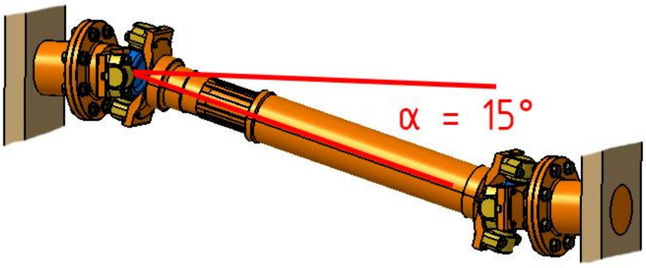

Fig. 6 *Cardan shaft sloped under angle 15° with parallel centerlines of the input and output shaft*

The graphical output of the angular acceleration obtained from the central cardan shaft proves cardan error because the angular acceleration is not zero and the shape of the curve is sinusoidal. The sinusoidal curve shape in Fig. 7 shows two areas with acceleration and two areas with deceleration during one revolution of the input driving shaft.

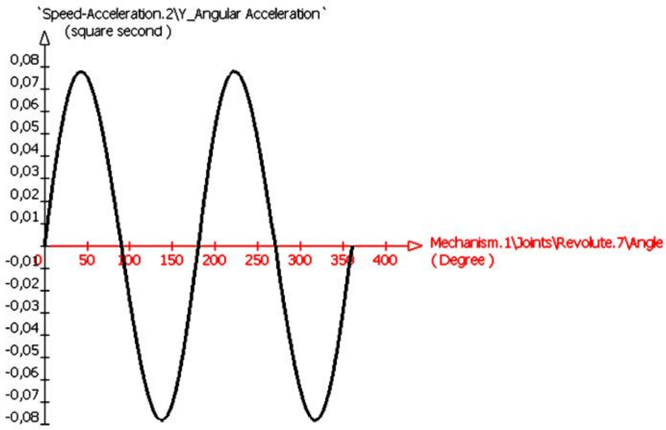

Fig. 7 *Angular acceleration central cardan shaft in dependence on rotation angle of the input driving shaft (alpha 15° and 1 turn of input driving shaft)*

Angular acceleration for the output driven shaft is shown in Fig. 8, where can be clearly seen that angular acceleration during a complete turn of the input driving shaft constant with a zero value. This result obtained through kinematic simulation using the CAD/CAM/CAE system CATIA V5 proves, that using two universal joints in cardan shaft construction with parallel input and output shafts leads to neutralization of cardan error and input and output angular speeds and accelerations for both shafts are constant and equal zero. This theory was confirmed through equations [8].

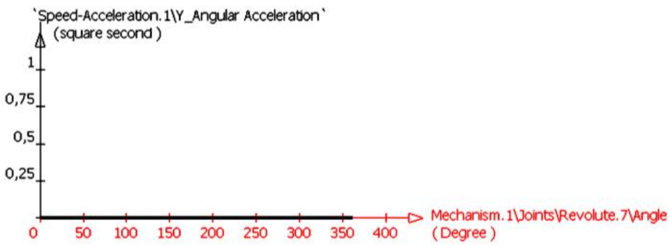

Fig. 8 *Angular acceleration of output driven shaft in dependence on the rotation angle of the input driving shaft*

The second case completed on the same cardan shaft construction but with the central cardan shaft was sloped under a higher angle with the value 25°.

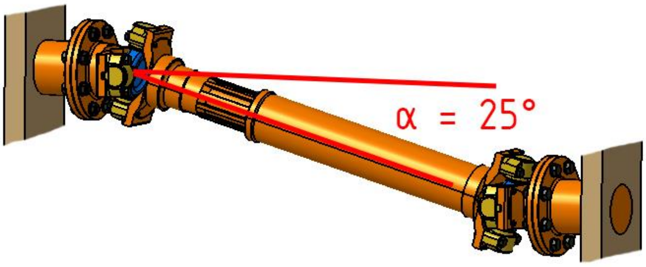

__Fig. 9__ Cardan shaft sloped under angle 25° with parallel centerlines of the input and output shaft

The graph in Fig. 9 shows the obtained values of angular acceleration of the central cardan shaft where it can be seen that the amplitude of angular acceleration has a higher value than in Fig. 7. When the angle α between the input driving shaft and central cardan shaft (central cardan shaft and output driven shaft) grows, then the amplitude of angular acceleration grows too.

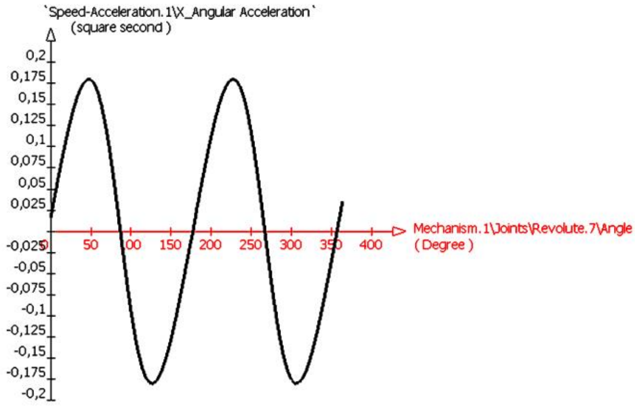

__Fig. 10__ Angular acceleration of the central cardan shaft in dependence on rotation angle of the input driving shaft (alpha 25° and 1 turn of input driving shaft)

The third case simulates the variable cardan shaft where the angle α changes value from -40° to 40° while the input driving shaft did four turns (1440°).

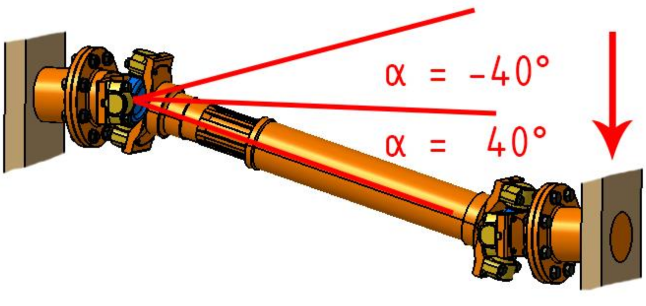

__Fig. 11__ Cardan shaft with variable slope angle of central shaft in range from -40° to 40°

Fig. 12 shows angular acceleration during the whole kinematic analysis. This type of analysis also proves that if angle α is decreasing then angular acceleration is decreasing too and vice-versa.

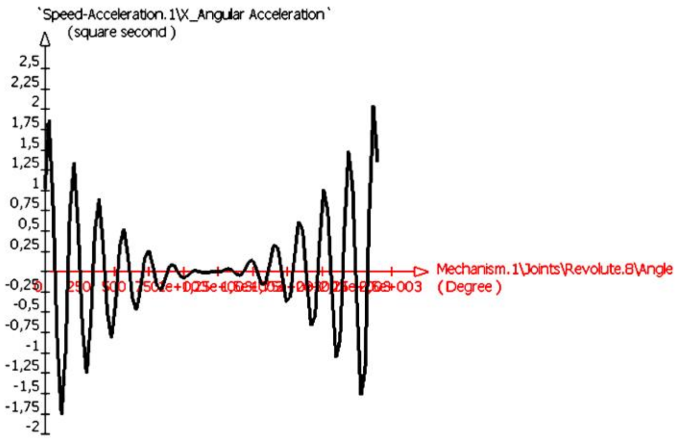

Fig. 12 *Angular acceleration of the central shaft in dependence on rotation angle of driving input shaft (alpha from -40° to 40° and 4 turns of input driving shaft)*

DISCUSSION

If we want to use a cardan shaft for higher axis deviation then we should use a design with two universal joints as you can see in Fig. 13b. With this solution, there are two deviation axis α_1 and α_2 which generate two cardan errors of the same size but opposite direction.

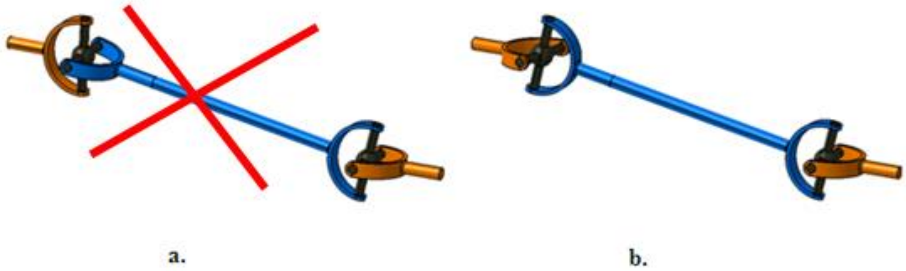

Fig. 13 *Cardan shaft assembled from two universal joints (a- incorrect, b- correct)*

CONCLUSION

The results from kinematic simulation obtained through the CAD/CAM/CAE system CATIA V5 proves theory analytically obtained equations.

The use of one universal joint will lead to an angular velocity and acceleration with sinusoidal shape on the driven shaft as can be seen in Figs. 7, 10 and 12. This state is also valid for a central cardan shaft, when the cardan shaft is assembled from two universal joints.

By using two universal joints, a stabilized rotational speed is achieved at the output driven shaft (Fig. 13b). If the cardan shaft is assembled incorrectly as you can see in Fig. 13a, then the angular speed and acceleration of the output driven shaft will be affected by two cardan errors. This state is very bad for the whole machine because vibrations caused by incorrectly assembled cardan shaft can lead to permanent failure of the machine.

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