TORSIONAL VIBRATIONS OF ELECTROMECHANICAL SYSTEMS

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Abstract

Non-linear electromechanical system consisting of DC motor and mechanical gearbox with gearing clearance is solved. The existence of clearance and possibility of development of transient phenomena make it unnecessary to give more accuracy to linear models. The resisting torque of elastic couplings is introduced as non-linear function.

Introduction

A machine aggregate (Fig.1a) is a dynamic system consisting as a rule from a driving machine gearing (reducing) mechanism with its binding, controlling and commanding accessories and a driven plant. The system characteristics, as well as characteristics of the individual subsystems are a result of their mutual accouplement and interference during operational activity. The system characteristics depend not only on the subsystems' initial characteristics, on their depreciation and overall status within the relevant time, but mainly on external phenomenon valid for the operating time of the system.

From point of gearing a lot of factors influence on the dynamics of the machine aggregate. The internal ones are mainly the choice of gearing basic parameters, material, production technology, accuracy of toothing and gearing clearance, but deformations, production accuracy, wear and working environment.

Non-linear electromechanical system consisting from direct-current motor and mechanical gearbox with gearing clearance is solved. Articles [1], [2] treat with peculiarities of mechanical subsystem of an electric drive, excited by kinematic and geometric inaccuracies of gearing and by their clearances. The analysis shows, that after taking kinematic and geometric inaccuracies into consideration, a gear ratio becomes a variable of displacement angle with the frequency proportional to the motor speed. A parametric excitation in elastic systems is caused by this [3] and a rise of dynamic load of the machine aggregate too. Presence of a clearance and a possible transients described in [4] demand for cases of small deviations from a point of static balance to create much more precise linear models than those used in articles quoted therein before [5], [6].

Mechanical and mathematical models

A mathematical model allows to study steady state and resonant phenomena in an electromechanical dynamical system consisting of an independently excited DC electric motor and a gearing, considering the same gear ratio and gearing clearance as in [2], [4].

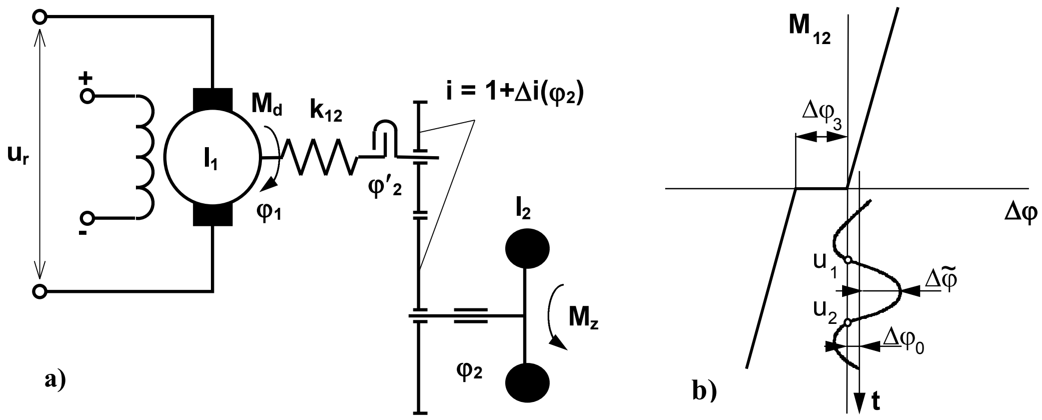


Fig. 1 *Dynamical models of machine aggregate with gearing*
a) *electromechanical system*, b) *dependence* $M_{12} = F(\Delta\varphi)$

Motion model of the system (Fig.1a) is described by motion equations of the mechanical subsystem

$$
\begin{aligned}
M_d - M_{z1} - M_{12} &= \beta T_{M_1} \frac{\mathrm{d}\omega_1}{\mathrm{d}t}, \\
M_{12} - M_z &= \beta(\mu - 1) T_{M_1} \frac{\mathrm{d}\omega_2}{\mathrm{d}t},
\end{aligned}
\tag{1}
$$

and by a dynamic torque equation of the electric motor

$$
\tau_e \frac{\mathrm{d}M_d}{\mathrm{d}t} + M_d = \beta(\omega_0 - \omega_1), \tag{2}
$$

where

ω_1, ω_2 - mechanical angular speeds of the electromotor rotor (driving shaft) and the driven shaft,
φ_1, φ_2 - angular displacement of the shafts,
ω_0 - angular no load speed of the electromotor (of the driving shaft)
M_{12} - elastic coupling torque,
M_d - driving (internal) electromagnetic torque generated by the rotor of the electric motor (by the driving shaft),
M_z - load torque,
M_z - friction torque,
β - static characteristic stiffness coefficient,
k_{12} - stiffness of elastic coupling,
τ_e - DC electromotor electromagnetic time constant,
$T_{M1} = I_1/\beta$ - electromechanic time constant of the driving shaft,
$\mu = (I_1 + I_2)/I_1$ - ratio of total reduced (torque of) inertia and driving shaft inertia (DC motor rotor + tightly coupled parts),
I_1, I_2 - reduced inertia of the driving and driven shaft.

In this case the elastic coupling torque M_{12} is a non-linear function $F(\Delta\varphi)$ of difference of angular displacement $\Delta\varphi = \varphi_1 - \varphi_2'$ (Fig.1b), where $\varphi_2' = \varphi_2 + \Delta\varphi_{\max} \sin \omega t$, [2], [4].

Supposing $\mu > 5$ all the dynamic processes are close to the one-mass system process [2] and φ_2 = const., mass of the plant (loading mechanism) has no influence to system oscillation [6].

Amplitude-frequency characteristics

To analyse a steady state motion of the forced oscillation we shall suppose that $\Delta\varphi$ on the non/linear element input is changed sinusoidally with exciting frequency and has constant component equal to average load torque (Fig. 1b).

Block diagram of the machine aggregate is shown in Fig. 2.

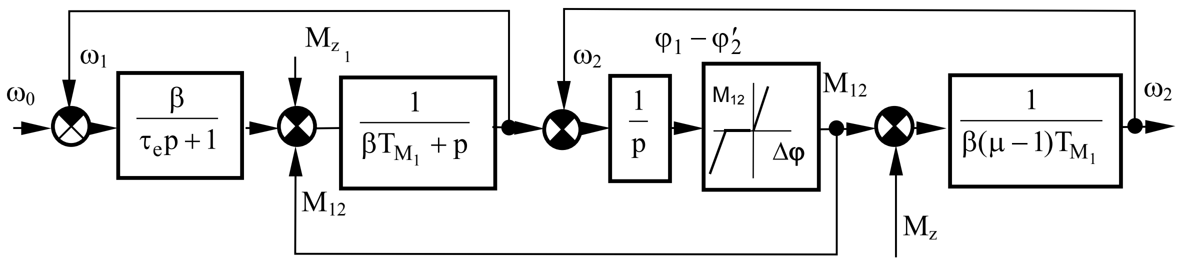


Fig. 2 *Block diagram of the machine aggregate*

The motion equation in the Laplace transform has the following form [6]

$$(I_1\tau_e p^3 + I_1 p^2 + \beta p)\Delta\varphi(p) + (1+\tau_e p)F(\Delta\varphi) = \\ = M_\phi - (\beta p + I_1 p^2 + I_1\tau_e p^3)\Delta\varphi_2(p), \tag{3}$$

where

$\Delta\varphi_2(p) \cong \Delta\varphi_2(t) = \Delta\varphi_{max}\sin\omega t$ - excitation induced by the difference of angular deviation in gearing [4],

$\Delta\varphi_{max}$ - maximal difference of angular deviation [4-6],

ω - frequency of change of angular deviation,

$M_\phi = \dfrac{cu_r}{R_{r\Sigma}} - \beta\omega_2$ - average value of the load torque,

$\tau_e = L/R_{r\Sigma}$ - electromagnetic time constant of the driving DC motor,

L - magnetic induction of DC motor armature,

u_r - DC motor armature (rotor) supply voltage,

$R_{r\Sigma}$ - total resistance of the DC motor armature circuit,

$c = k\Phi$ - DC motor constant,

Φ - magnetic flux in the DC motor,

$p = \mathrm{d}/\mathrm{d}t$ - differential operator, Laplace operator.

To analyse a steady state motion of the forced oscillation we shall suppose that $\Delta\varphi$ on the non-linear element input is changed sinusoidally with exciting frequency and has constant component equal to average load torque. The solution of the (3) is supposed to be in the form

$$\Delta\varphi = \Delta\varphi_0 + A\sin\omega t = \Delta\varphi_0 + \Delta\tilde{\varphi}, \tag{4}$$

where

A - amplitude of periodic component of angular deviation,
$\Delta\tilde{\varphi}$ - periodic component of angular deviation.

The non-linear function $F(\Delta\varphi)$ is replaced by an approximation from the harmonic linearization

$$F(\Delta\varphi) = q_0(A, \omega, \Delta\varphi_0) + q_1(A, \omega, \Delta\varphi)\Delta\tilde{\varphi}, \tag{5}$$

where q_0, q_1 are coefficients of the harmonic linearization. For $u = \omega t$ with respect to Fig.1b they have a form

$$\begin{aligned} q_0(A, \Delta\varphi_0) &= \frac{1}{2\pi}\int_{u_1}^{u_2} k_{12}(\Delta\varphi_0 + A\sin u)du = \\ &= \frac{k_{12}A}{\pi}\sqrt{1-\left(\frac{\Delta\varphi_0}{A}\right)^2} - \frac{k_{12}}{2}\Delta\varphi_0 + \frac{k_{12}\Delta\varphi_0}{\pi}\arcsin\frac{\Delta\varphi_0}{A}, \end{aligned} \tag{6}$$

$$\begin{aligned} q_1(A, \Delta\varphi_0) &= \frac{1}{\pi A}\int_{u_1}^{u_2} k_{12}(\Delta\varphi_0 + A\sin u)\sin u du = \\ &= \frac{k_{12}}{2} - \frac{k_{12}}{\pi}\arcsin\frac{\Delta\varphi_0}{A} - \frac{k_{12}\Delta\varphi_0}{\pi A}\sqrt{1-\left(\frac{\Delta\varphi_0}{A}\right)^2}. \end{aligned} \tag{7}$$

The solution of (3) after substitution of the function (5) can be written in a form of two equations for the constant and the periodical component

$$\frac{k_{12}A}{\pi}\sqrt{1-\left(\frac{\Delta\varphi_0}{A}\right)^2} - \frac{k_{12}\Delta\varphi_0}{2} + \frac{k_{12}\Delta\varphi_0}{\pi}\arcsin\frac{\Delta\varphi_0}{A} = M_\phi, \tag{8}$$

$$\begin{aligned} &(I_1\tau_e p^3 + I_1 p^2 + \beta p)\Delta\tilde{\varphi}(p) + (1+\tau_e p)\left[\frac{k_{12}}{2} - \frac{k_{12}}{\pi}\arcsin\frac{\Delta\varphi_0}{A} - \right. \\ &\left. - \frac{k_{12}\Delta\varphi_0}{\pi A}\sqrt{1-\left(\frac{\Delta\varphi_0}{A}\right)^2}\right]\Delta\tilde{\varphi}(p) = -(\beta p + I_1 p^2 + I_1\tau_e p^3)\Delta\varphi_2(p), \end{aligned} \tag{9}$$

or in a non-dimensional form [6]

$$\frac{A_*}{\pi}\sqrt{1-\left(\frac{\Delta\varphi_{*0}}{A_*}\right)^2} - \frac{\Delta\varphi_{*0}}{2} + \frac{\Delta\varphi_{*0}}{\pi}\arcsin\frac{\Delta\varphi_{*0}}{A_*} = M_{*\phi}, \tag{10}$$

$$\left(\frac{\tau_e}{\omega_0^2}p^3 + \frac{1}{\omega_0^2}p^2 + \left(q\tau_e + \frac{1}{T_{M_1}\omega_0^2}\right)p + q\right)\tilde{M}_{*12}(p) = -\left(\frac{1}{T_{M_1}\omega_0^2}p + \frac{1}{\omega_0^2}p^2 + \frac{\tau_e}{\omega_0^2}p^3\right)\Delta\tilde{M}_*(p), \tag{11}$$

where

$$q = \frac{1}{2} - \frac{1}{\pi}\arcsin\frac{\Delta\varphi_{*0}}{A_*} - \frac{\Delta\varphi_{*0}}{\pi A_*}\sqrt{1-\left(\frac{\Delta\varphi_{*0}}{A_*}\right)^2}, \tag{12}$$

$\widetilde{M}_{*12}=\dfrac{k_{12}\Delta\widetilde{\varphi}(t)}{M_b}$ - ratio of the first harmonic of the elastic coupling torque and the base moment M_b

$$\Delta M_* = \frac{k_{12}\Delta\varphi_2(t)}{M_b} = \frac{k_{12}\Delta\varphi_{\max}}{M_b}\sin\omega t = \Delta M_{*\max}\sin\omega t, \qquad (13)$$

$A_* = \dfrac{k_{12}A}{M_b} = \dfrac{A}{\Delta\varphi_b} = \widetilde{M}_{*12\max}$ - amplitude of the periodical component in the non-dimensional form,

$\Delta\varphi_{*0} = \Delta\varphi_0/\Delta\varphi_b$ -constant component of solution (4),

$\Delta\varphi_b = M_b/k_{12}$ - basic angular displacement,

$\omega_0 = \sqrt{\dfrac{k_{12}(I_1+I_2)}{I_1 I_2}}$ - natural angular velocity,

For analysis of the system steady state motion with clearance we obtain

$$A_{M_{*12}} = \frac{\widetilde{M}_{*12\max}}{\Delta M_{*\max}} = \sqrt{\frac{\tau_e\omega^2\left(\dfrac{1}{\tau_e T_{M_1}\omega_0^2}-\eta^2\right)+\eta^4}{(q(A_*,M_{*\phi})-\eta^4)^2+\tau_e^2\omega^2\left(q(A_*,M_{*\phi})+\dfrac{1}{\tau_e T_{M_1}\omega_0^2}-\eta^2\right)^2}}, \qquad (14)$$

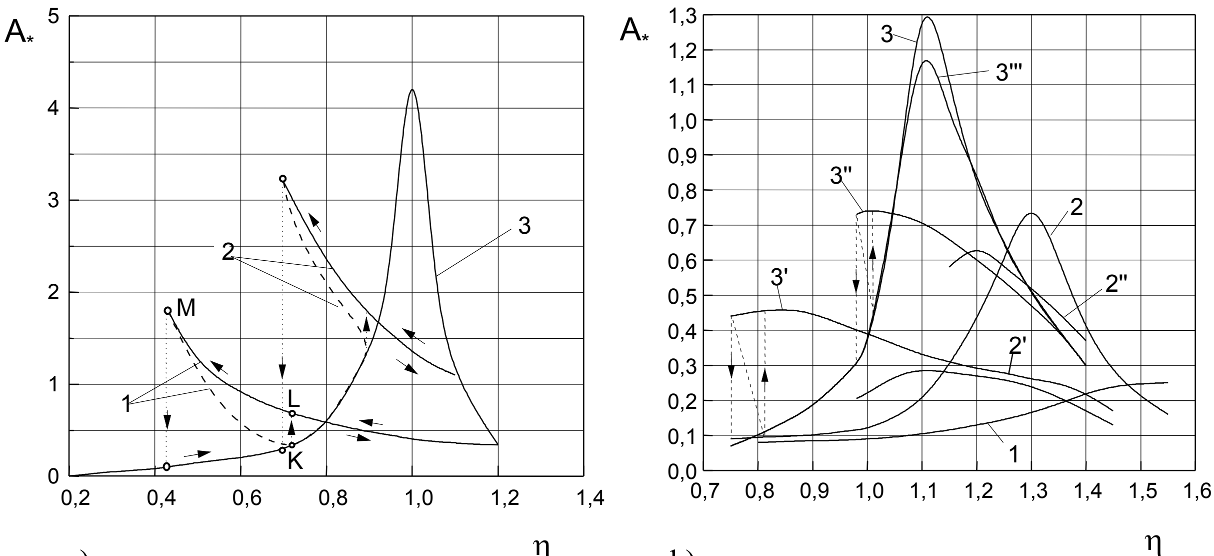
where $\eta = \omega/\omega_0$ - frequency ratio.

From (14) results that in the nonlinear system under consideration the vibration amplitudes at given excitation frequency depend not only on excitation ΔM_* but also on the average value of the load $M_{*\Phi}$.

Results and analysis of results

In the Fig. 3a can be seen that non-linearity caused by gearing clearance results in a considerable deformation of the resonance curve and the resonance is manifested already in lower frequencies than the resonance frequency and shows an ambiguity of disturbances amplitude. Also, a specific property of a system with clearance, a dependence on the $M_{*\Phi}$ can be seen.

The curve No.3, Fig.3a, presents an amplitude-frequency characteristics of a linear system [1]. At a given speed of the electric motor, greater than zero, both frequency ω and the corresponding variable component of gear load increase. If the variable component is less than the average value of gear load, impact of clearance is not manifested (it is closed) and the system plays to be linear. For $M_{*\Phi}$ = 0.2 (curve 1) in the point K stability occurs, i.e. $A_* = \Delta M_{*\Phi}$. If the amplitude keeps increasing, the clearance becomes open. Due to this fact the harmonic linearized stiffness decreases and the area of resonance is shifted to the point K. Consequently, the amplitude for the same frequency increases, coefficient of harmonic linearization decreases again - it means that an avalanche increase of amplitude to the point L, curve 1 is generated. Balance is reached in the point L again. For decreasing frequency ω the amplitude grows up to point M, where the maximal amplitude at given frequency, i.e. $A_* = f(q)$ is less than $q = f(A_*)$ at $M_{*\Phi}$ = 0.2. Conditions for creation of resonance are corrupted and values of amplitude fall down below values given by the curve 3.



a) b)

***Fig. 3** Amplitude-frequency characteristics*
a) *low damping system with gearing clearance*, b) *high damping system with clearance*

Increase of the average load value brings about an increase of amplitude too (Fig.3a - curve 2) and the character of deformation is the same as in the previous case. From the Fig.3a can also be seen that in the zone of linear resonance the non-linearity caused by gearing clearance limits vibration amplitude values to values close to average load value, but expands the zone of resonance vibrations to the lover frequencies more than the average load.

The influence of damping effect from the electrical drive to the steady state motion in the system with a clearance are given by curves 1, 2, 3 in the Fig.3b, for a linear system with $\omega_0 = 22$ rad/s and the time constants τ_e, T_{M_1} as follows:

- curve No 1: $\tau_e = 0.03$ s, $T_{M_1} = 0.03$ s;
- curve No 2: $\tau_e = 0.1$ s, $T_{M_1} = 0.03$ s;
- curve No 3: $\tau_e = 0.1$ s, $T_{M_1} = 0.06$ s.

The damping for parameters corresponding to the curve 3 is attenuated, hence the calculated relations $A_* = f(\eta)$ for $\Delta M_{*\max} = 0.15$ and $M_{*\Phi} = 0.1$ (curve 3') or $M_{*\Phi} = 0.4$ (curve 3''), or $M_{*\Phi} = 1.0$ (curve 3''') have lower amplitudes.

Higher damping cause not only lower amplitudes, but also decreases of non-linear resonance (curve 2' for $M_{*\Phi} = 0.1$ and curve 2'' for $M_{*\Phi} = 0.4$). For the maximal damping value curves of non-linear resonance are in fact associated with the characteristics of a linear system (curve 1). Choice of optimal parameters for a linear system minimizes the possibility of vibration in the system with gearing clearance.

Conclusions

The method of harmonic linearization is an effective mean to analyze physical peculiarities of steady state vibrating processes in nonlinear electromechanical systems considering the real conditions for the system motion with variable gear ratio and a clearance [2], [3], [5], [6]. Using the method of harmonic linearization it is to respect the fact that it only gives result close to the reality. If the characteristic equation for (3) has the form of

$$Q(p)\Delta\varphi(p) + R(p)F(\Delta\varphi) = 0$$

formulation of conditions characterizing the accuracy of the method is as follows:

1. The rate of polynomial $R(p)$ must be lower than the rate of polynomial $Q(p)$,
2. The polynomial $Q(p)$ must have neither complex conjugate roots nor positive real roots, the relation $q(A)$ must be smooth.

The harmonic linearization allows to explain character of processes and physical properties of a nonlinear subject. Quantitative differences in computation of resonance amplitudes rated 30-40% can be explained by specific effects of internal excitations (disturbance) caused by pulsating gear ratio. The relation (3) is derived considering the exciting torque (13) to influence the rotor of the DC electric motor and at the origin of clearance is interrupted. This fact decreases the precision of quantitative evaluations, but allows to solve successfully tasks of properties analysis in nonlinear electromechanical systems.

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