MODIFICATION OF DYNAMICAL PROPERTIES OF CIRCULAR DISCS BY STRESS FIELDS

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Introduction

The circular discs are structural elements widely used in the structural and processing applications. One of the most used geometric shape for processing and cutting operation material is thin circular disc - circular saw blade. Circular saw blade are widely used for cutting and forming metal and non-metal materials.

The quality of the cutting is mostly influenced by dynamical behaviour of circular saw blade in cutting process. The large transversal displacements of circular saw blade, induced by rotation of circular saw blade and external effects, occur during cutting process. In many cases, the loss of dynamical stability and arising of resonance state are caused by the reduction of natural frequencies of the disc due to the thermal compressive stress induced by cutting heat in the peripheral region. In dependence on cutting conditions, the temperature of cutting in the peripheral region increases up 500° C. It is necessary to eliminate these inconvenient effects by some design or technological treatments.

One of the technique of the disc modifications to achieve the required dynamic properties is to initiate pre-stress in disc plane. This is concerned primarily with "tuning" of dynamical properties of vibrating circular discs by technological treatments inducing the residual in-plane stresses. It is possible to obtain the disc in-plane stress either roll-tensioning of disc surface or volume transformation of disc part. In the roll-tensioning process, the disc is compressed within a certain annular contact zone between two opposing rollers. The contact zone of circular saw blade is plastically deformed and the residual stresses occur in whole disc plane. The effects of residual stresses induced by roll-tensioning on dynamical properties (natural frequencies, mode shapes) are analysed.

The natural frequency characteristics for various rolling position, various rolling depth of the annulus and for one resp. two rolled annular fields are obtained by modal analysis using Finite Element Method (FEM) - ANSYS software package. The role of residual stresses obtained by rolling is assessed from the change in natural frequencies and modal shapes.

Formulation of the problem

The circular saw blade, which is analysed, has the shape of circular disc. In the following, we consider an isotropic homogeneous circular disc of a outer radius R_0, inner radius r_0 and thickness h (Fig.1). The radius r_0 specifies a circle where is the disc clamped by flanges. The plastically deformed zone of is determined by inner radius r_1 and outer radius r_2.

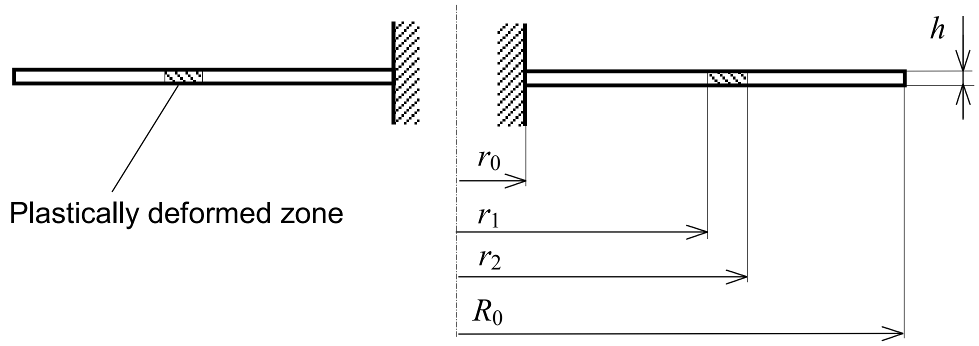

Fig.1 *The circular disc with plastically deformed zone*

The fundamental considerations and derivation of equations of motion are based on Kirchhoff's assumptions. The field of displacements in the cylindrical coordinates r, φ, z, using Kirchhoff plate theory can by written as

$$\begin{aligned} u_r(r,\varphi,z,t) &= u(r,\varphi) - z\frac{\partial w(r,\varphi,t)}{\partial r}, \\ v_\varphi(r,\varphi,z,t) &= v(r,\varphi) - \frac{z}{r}\frac{\partial w(r,\varphi,t)}{\partial \varphi}, \\ w_z(r,\varphi,z,t) &\cong w(r,\varphi), \end{aligned} \tag{1}$$

where $u(r,\ \varphi)$, $v(r,\ \varphi)$ and $w(r,\ \varphi)$ are displacement of point laying on neutral plane of the circular disc in coordinate directions.

The stress-strain relations in the cylindrical coordinates can be written

$$\boldsymbol{\sigma} = \mathbf{D}\boldsymbol{\varepsilon}, \tag{2}$$

where $\boldsymbol{\sigma}$ - stress vector,
$\boldsymbol{\varepsilon}$ - strain vector,
$\mathbf{D}$ - elasticity matrix.

Using the finite element formulation, the equation of motion for a free vibration of circular disc modelled by FEM is described by expression

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{0}, \tag{3}$$

where $\mathbf{M}$ - mass matrix,
$\mathbf{K}$ - stiffness matrix,

$\ddot{\mathbf{q}}$ - nodal accelerations,

$\mathbf{q}$ - nodal displacements.

Generally, after initiation of in-plane stress fields, the stress-strain relations with initial stress and initial strain are given by

$$\boldsymbol{\sigma} = \mathbf{D}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_0) + \boldsymbol{\sigma}_0, \tag{4}$$

where $\boldsymbol{\sigma}_0$ - initial stress vector,

$\boldsymbol{\varepsilon}_0$ - initial strain vector.

The equation of motion for a free vibration of in-plane stressed disc modelled by FEM is described by expression

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{K} + \mathbf{K}_\sigma)\mathbf{q} = \mathbf{0}, \tag{5}$$

where $\mathbf{K}_\sigma$ - stiffness matrix following from stress distribution induced by rolling.

The mass distribution of pre-stressed circular disc after rolling is not changed, but the residual stress distribution is developed in disc plane in consequence of the bending stiffness of disc varies greatly. Using equation (5), the natural frequencies and modal shapes of a circular disc with roll induced residual stress distribution can be obtain from the following eigen-value problem

$$(\mathbf{K} + \mathbf{K}_\sigma - \omega_i^2\mathbf{M})\boldsymbol{\psi}_i = \mathbf{0}, \tag{6}$$

where ω_i - natural frequency of the circular disc,

$\boldsymbol{\psi}_i$ - eigen-vector describing modal shape of the circular disc.

In order to calculate the variation of disc stiffness $\mathbf{K}_\sigma$ after rolling, we must know the residual stress distribution in a disc plane. To the determination of residual stress distribution, the method of thermal stress loading is used. The thermal expansion embedded to annular fields induces the stress distribution in circular disc which is analogous to the stress distribution initiated by roll-tensioning.

The natural frequencies and modal shapes of circular disc with residual stress distribution for different rolling position, depth and width of rolled annulus can be determined from calculation model which is described by equation (6). To obtain optimal rolling radius, it is necessary to determine the natural frequency curves for the various position of center radius of the rolled annulus $r_c = (r_2 + r_1)/2$ (r_1 - inner radius, r_2 - outer radius of rolled annulus, see Fig.1).

Numerical example and simulations

We consider a circular disc of the outer diameter 240 mm, thickness 1.8 mm and flange diameter is 50 mm. The disc for analysis is assumed to be perfectly fixed in region $r \le r_0$ (see Fig.1). The outer edge of disc is free. The depth of the rolling is varied and for simulation are used values 1 μm, 2 μm, 3 μm and 4 μm. The effect of the position and depth of roll-tensioning annular field and number of annular fields on natural frequencies is investigated. The dynamical properties of the disc were determined using FEM by software package ANSYS.

The distribution of radial and tangential in-plane residual stresses of circular discs with one roll-tensioning annular field (for example r_c = 0.05 m) is shown on Fig. 2. The width of plastically deformed annular field is $r_2 - r_1$ = 10 mm and depth is Δz = 1 μm

The distribution of radial and tangential in-plane residual stresses of circular discs with two roll-tensioning annular fields (for example $r_{c1} = (r_{12} + r_{11})/2 = 0.03$ m, $r_{c2} = (r_{22} + r_{21})/2 = 0.07$ m; r_{ci} is radius of the i-th annular fields, r_{i1}, resp. r_{i2} are inner, resp. outer radius of the i-th annular fields) is shown on Fig. 3. The width of both plastically deformed annular fields is 10 mm and depth is $\Delta z = 1$ μm.

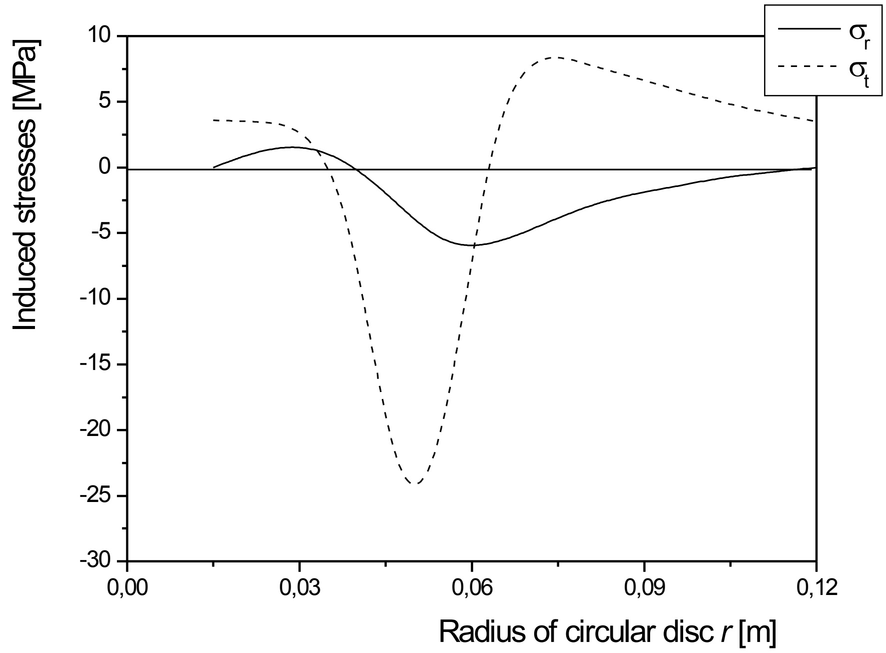

Fig. 2 *Distribution of radial and tangential stresses of circular disc with one pre-stressed annular field* ($r_c = 0.05$ m, $\Delta z = 1$ μm)

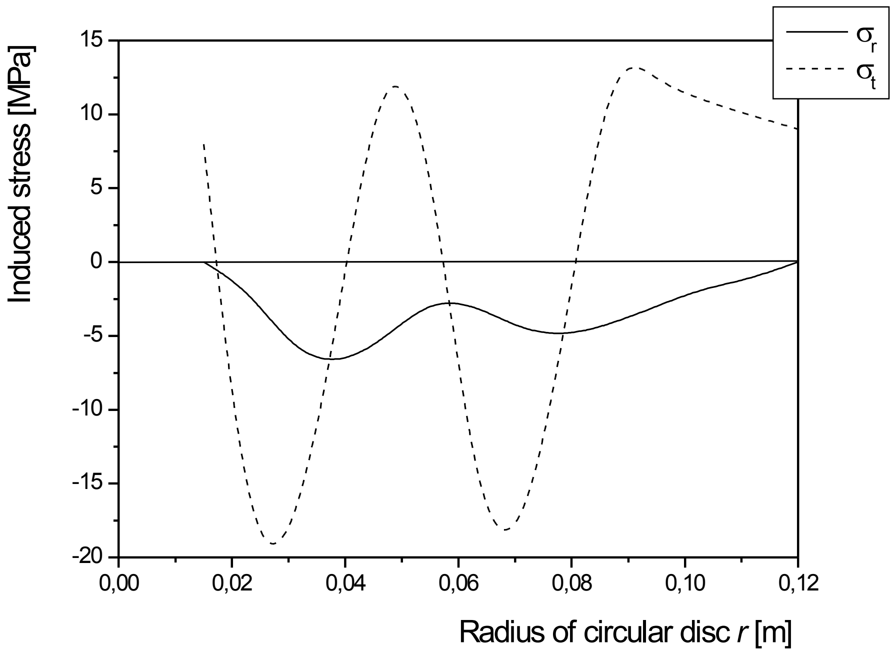

Fig. 3 *Distribution of radial and tangential stresses of circular disc with two pre-stressed annular fields* ($r_{c1} = 0.03$ m, $\Delta z_1 = 1$ μm; $r_{c2} = 0.07$ m, $\Delta z_2 = 1$ μm)

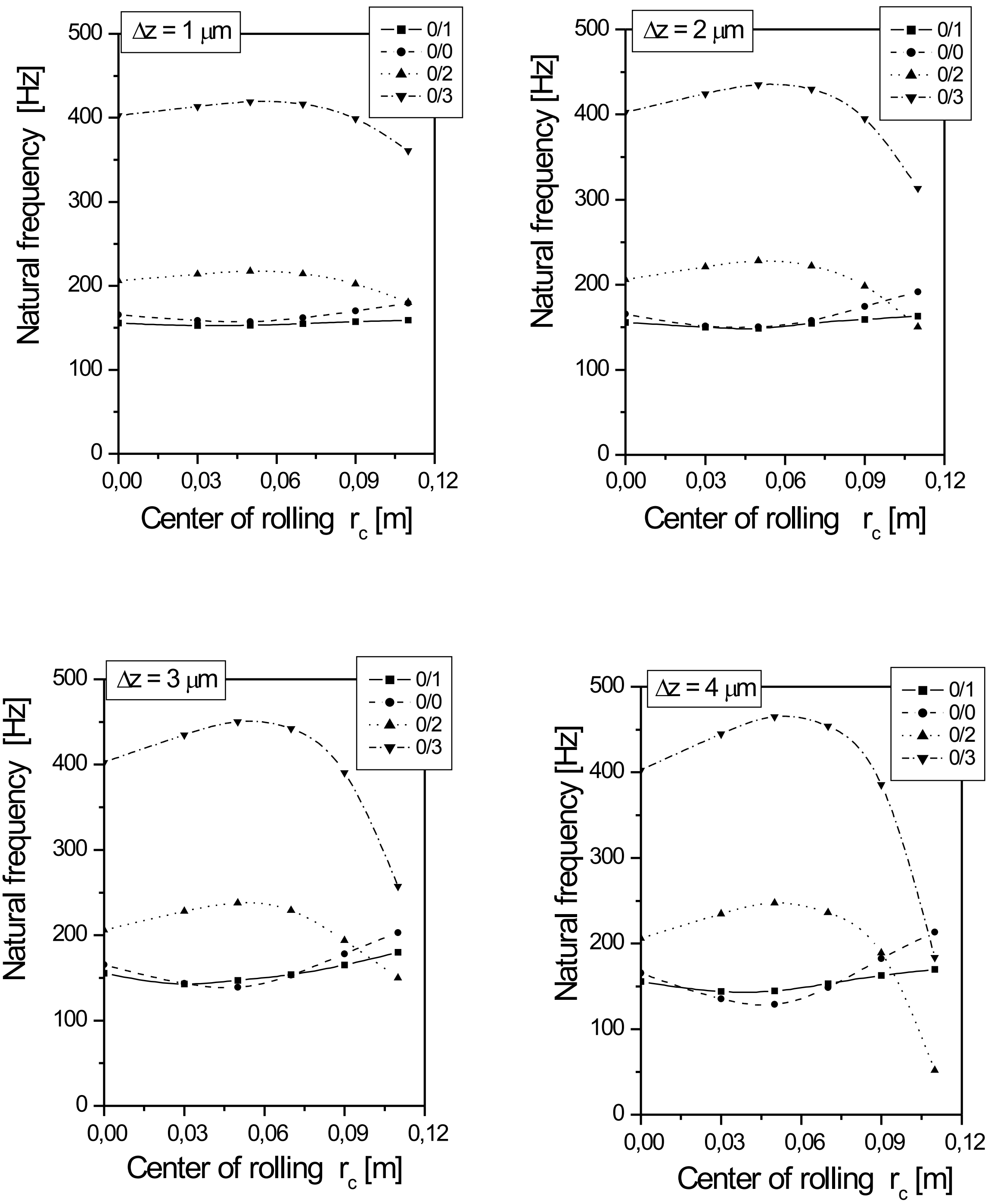

Fig. 4 *Natural frequency vs. center of rolling* r_c *for different depth of rolling* Δz
(*circular disc with one pre-stressed annular field*)

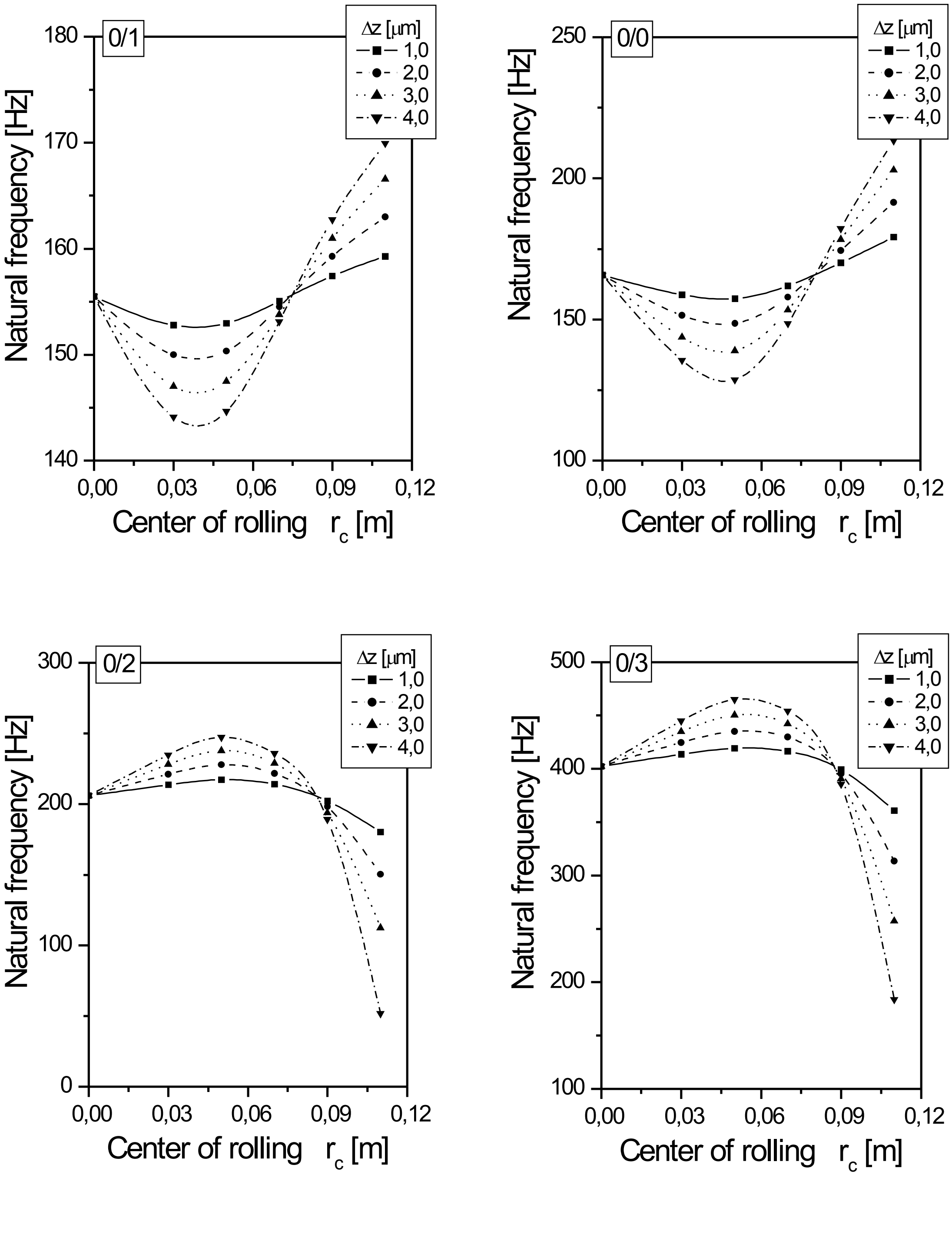

Fig. 5 *Natural frequency vs. center of rolling for different modal shapes* (*circular disc with one pre-stressed annular field*)

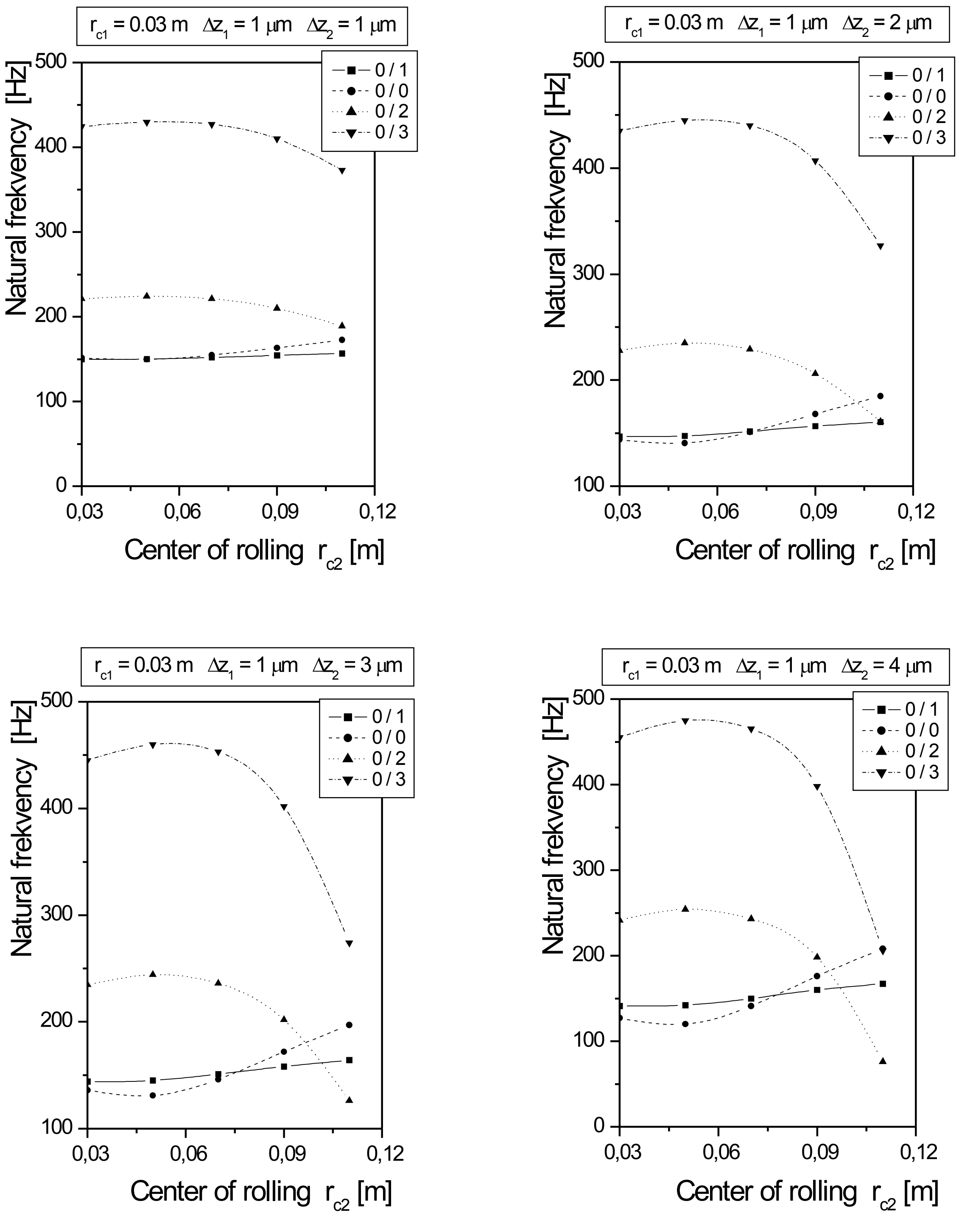

Fig. 6 *Natural frequency vs. center of rolling for different modal shapes*
(*circular disc with two pre-stressed annular fields* r_{c1} = 0.03 m, r_{c2} = 0.03 ÷ 0.12 m)

Fig. 4 ÷ 5 show the natural frequency curves (circular disc with one pre-stressed annular field) for modal shapes 0/1, 0/0, 0/2, 0/3 (number of nodal circles/number of nodal lines) calculated by FEM when r_c varies from 0.03 m to 0.11 m. The natural frequencies of circular disc before rolling are marked by r_c = 0.0 m. The tendency of curves for modal shapes 0/1 and

0/0 differs from curves for modal shapes 0/2 and 0/3. The natural frequencies of the modal shapes 0/2 and 0/3 increase with r_c until the maximum values near r_c = 0.06 m are reached; then they decrease. Contrary to this, the natural frequencies of the modal shapes 0/1 and 0/0 decrease with r_c and for r_c = 0.05 m reach the minimum; then they increase.

The natural frequency curves of circular disc with two pre-stressed annular fields (r_{c1} is constant value 0.03 m and r_{c2} varies from 0.03 m to 0.12 m) are shown on Fig. 6. The effect of two pre-stressed annular fields on dynamical properties of disc is similar as for disc with one pre-stressed field. Comparing of results, we obtain that the values of natural frequencies are different and position of extremes are shifted.

The effect of thermal compressive stress induced by cutting heat in the peripheral region we can see for r_c = 0.11 m, where natural frequencies of the modal shapes 0/2 and 0/3 for different depth of rolling are lower then natural frequencies of the modal shapes 0/1 and 0/0. For this case, the resonance of circular disc arises for lower frequencies. The reason why natural frequencies are varied due to rolling is that stiffness characteristics within the disc are locally changed by stress distribution.

Conclusions

The theoretical formulation and calculation model for analysis of dynamical properties of circular disc with residual stress distribution are presented. Finite element analysis for estimating of natural frequencies was used. For certain rolling radius r_c, the natural frequencies of modal shapes 0/2 and 0/3 become smaller than those before rolling and the purpose of tension stresses cannot be achieved. Therefore, the appropriate rolling position is necessary to determine from natural frequency characteristics calculated for various center of rolling.

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