

CHOOSING THE MOST APPROPRIATE MATHEMATICAL MODEL TO APPROXIMATE THE ABBOTT CURVE

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Abstract

The paper deals with the various mathematical models of Abbott-Firestone curve determination and choice of the best model to describe Abbott-Firestone curve equation.

Key words

surface roughness, approximation, mathematic models, Abbott-Firestone curve

Introduction

Each design drawing of part consists important data among with dimensions for proper function and life of part. One of them is surface characteristic – roughness.

Surface roughness is determined by trails left by cutting edge of tool. Method of surface measurement and evaluation is normalized by international ISO standards, based on profile method, evaluates the size of the surface profile – line originated from cutting the actual surface by defined area. Terms, definitions and parameters of surface character establish the standard STN EN ISO 4287:1999. In this standard is surface roughness pictured as protrusions of profile „Zp“ and depressions of profile „Zv“ in transverse direction (Fig. 1).

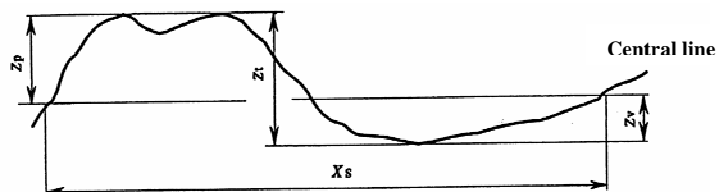


Fig. 1. Profile element [11]

Designer in drawing often prescribes the desired value of the median arithmetic deviation of roughness R_a . Nevertheless, it appears that such a prescription of roughness of the surface may not be sufficient.

Abbott-Firestone curve

Roughness, or bearing capacity can be characterized by the so-called share profile curve of the material, also called the Abbott curve, in the literature we can also find its name as Abbott-Firestone curve and in earlier standards is marked as „bearing surface curve”.

Material roughness share of profile $Rmr(c)$ is defined as the carrier length ratio of the profile to evaluated length (Fig. 2).

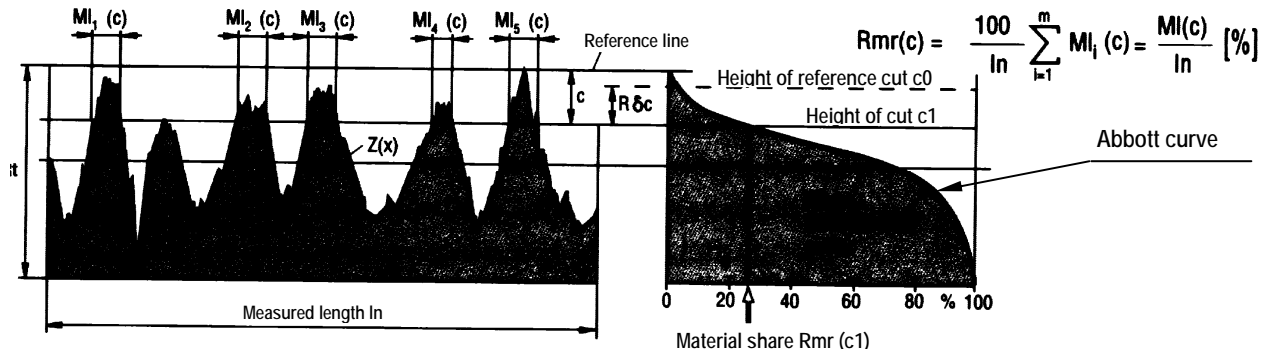


Fig. 2. Material ratio of the profile roughness [7]

In evaluation of the surface is used the graphical interpretation of the Abbott curve, which graphically describes the distribution of material in the length range of profile. We get it as the sum of the intersections of profile cuts in abscisses parallel to median profile line. The curve starts from the highest projection, where is 0% of the material. It ends in the lowest depression, where is 100% of the material. (Fig. 3)

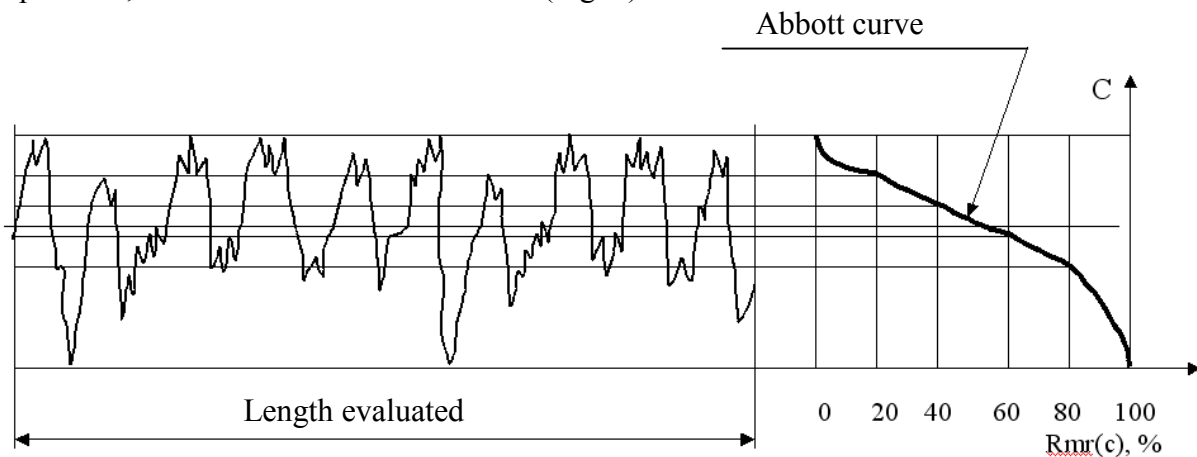


Fig. 3. Abbott curve

Abbott - Firestone curve of the actual surface is not a simple curve and its description by mathematical functions is very variable. If we put line j in the middle part of the Abbott curve, which best describes this part of the Abbott curve, it can be distinguished to three parts (Fig. 4) [8]:

- *Projection part*, line $(0,0), (x_2,z_2)$: this contains a summary of the highest projections of the profile surface and generally it will disappear in the running-in operation (function) of part surface.

- *Central exploitation part*, line (x_2, z_2) , (x_3, z_3) : is the most important and it also determines the life of parts.
- *Low depression part*, line (x_3, z_3) , (l, R_z) : it is not available for the functioning of parts and serves for anchoring the lubricating film.

Central exploitation part is better considering to quality of surface, if is closer to axis of abscissae (x coordinate). The second is the axis of ordinates (coordinate z) and is located in the height of surface profile roughness. The third axis (coordinate y) situates position of surface roughness profile, if it's possible.

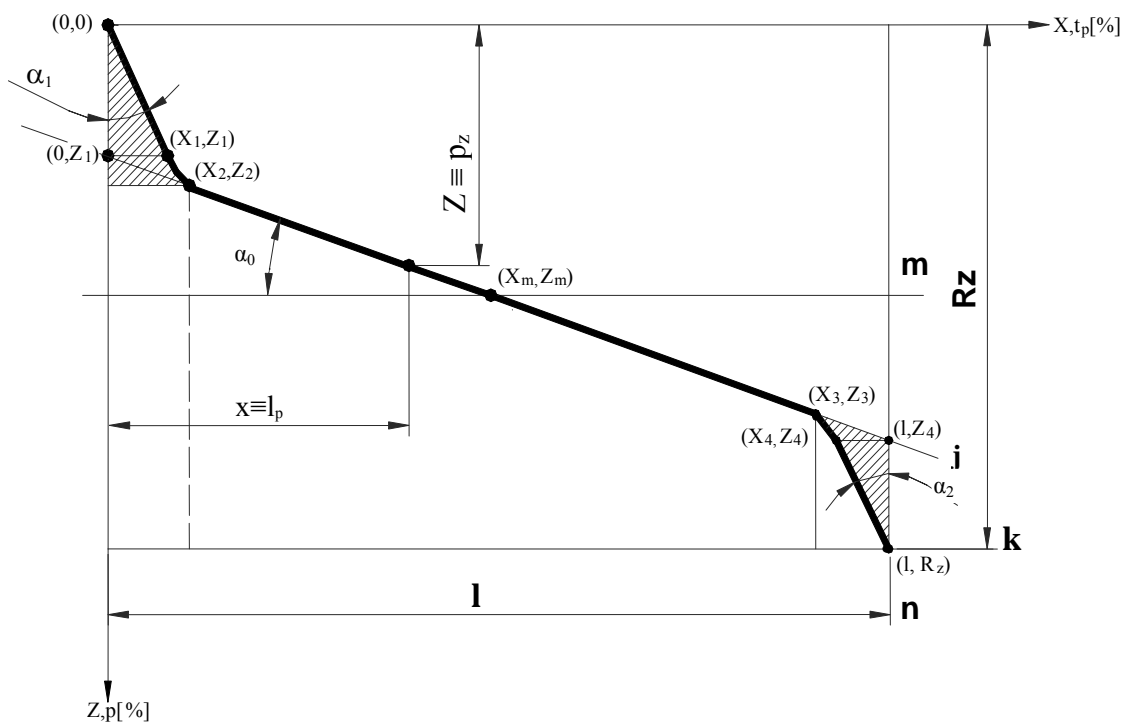


Fig. 4. Illustration of the Abbott curve [8]

Abbott-Firestone curve is a good characteristic for assessing the functional properties of surfaces and their possible exploitation. Can distinguish between different surfaces with the same value of R_a , or other height characteristics. Generally speaking, each type of surface is characterized by course of Abbott curve. Today's modern equipment for roughness measuring with the right software can evaluate and display the various properties of the surface (Fig. 5).

Abbott curves generated by computer give visualized concept about their general shapes, but they do not give a specific evaluation. Therefore, the Abbott curve approximation for assessing the quality is necessary.

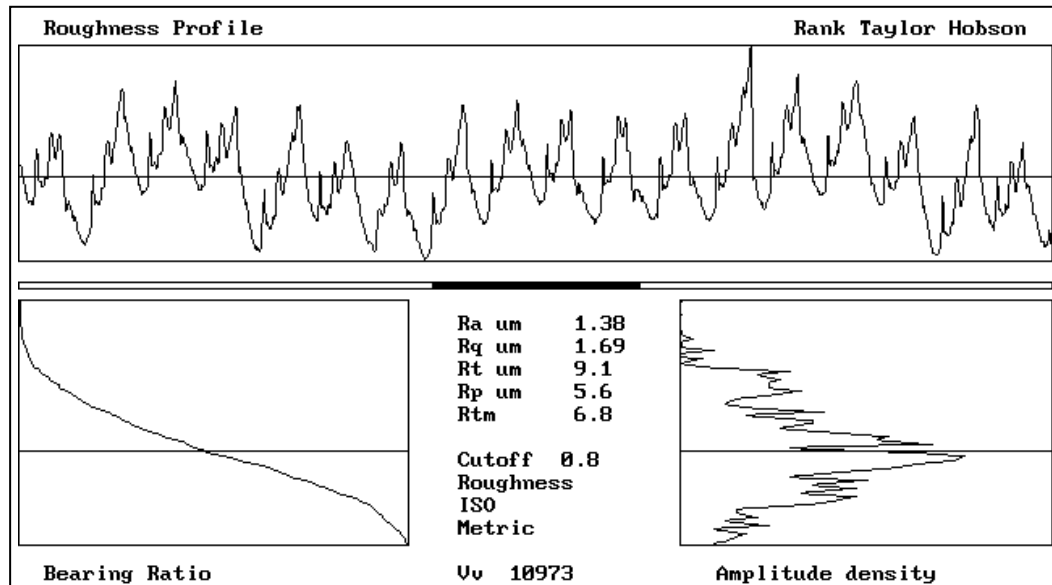


Fig. 5. Illustration of the Abbott curve PC output

Abbott curve approximation

Surface roughness is not one-dimensional problem. The parameter Ra is only the one height parameter. Considering that during the measurement of roughness, the profile roughness is most often captured in the anticipated direction of greater roughness, even one more (linear) roughness parameter must be prescribed or evaluated.

To find a suitable approximation of Abbott curve, we try to find approximate solutions to a problem that is less demanding than the exact solution. For mathematical expression is necessary to replace the curve expressed by function $f(x)$ for other more simple function $\phi(x)$, which we can easily express. We can find the approximate expression of curve shape - the central exploitation part. Trendlines can express features:

- **Linear function** - is the simplest expression of the measured variables. Values are expressed as a linear dependence of line. Linear function shows a situation in which something is increasing or decreasing at a constant rate.

$$y = a + bx, \tag{1}$$

where constant a express shifting lines compared to zero point, constant b is its slope.

- **Power function** – for Abbott curve approximation we can use power form

$$y = c \cdot x^n, \tag{2}$$

where x, z are axis, c and n are constants

Clearance by power function is used in case of increasing data values measured at intervals.

Dictating constants c and n for the power function can be difficult, because each change highly influences the shape of the Abbott curve and exponent changes very nature of the curve. [8], [9]

- **Exponential function**

$$x = c_1 \cdot e^{c_2 z} . \quad (3)$$

Abbott curve approximation as the exponential function is not easy.

- **Cubic parabola**

$$y = c \cdot x^3 . \quad (4)$$

The disadvantage of curve representation by cubic parabola is complexity compared to three-segment, eventually one-segment approximation.

- **Logarithmic function**

$$y = a \ln(x) + b . \quad (5)$$

Clearance by logarithmic function is used in the data that quickly rise or fall and gradually compensates. When translating data by logarithmic function, we can use both positive and negative values.

Abbott curve approximation by polynomial

Clearance by polynomial function is used for data, which vary and therefore they cannot be approximated by simpler functions. The degree of the polynomial can be given by the number of fluctuations in the data or by the number of curvature (maxima and minima) in the curve.

Abbott curve should be described somehow. The problem is that all possible approximation does not suit completely, therefore Abbott curve approximation by second or third degree seems to be most useful, namely:

$$x = c_0 + c_1 z + c_2 z^2 , \quad (6)$$

$$x = c_0 + c_1 z + c_2 z^2 + c_3 z^3 , \quad (7)$$

while the coefficients c_0 to c_3 should be prescribed in the design drawing. These approximations, however, were not sufficiently precise, therefore polynomial is chosen up to 9-th degree

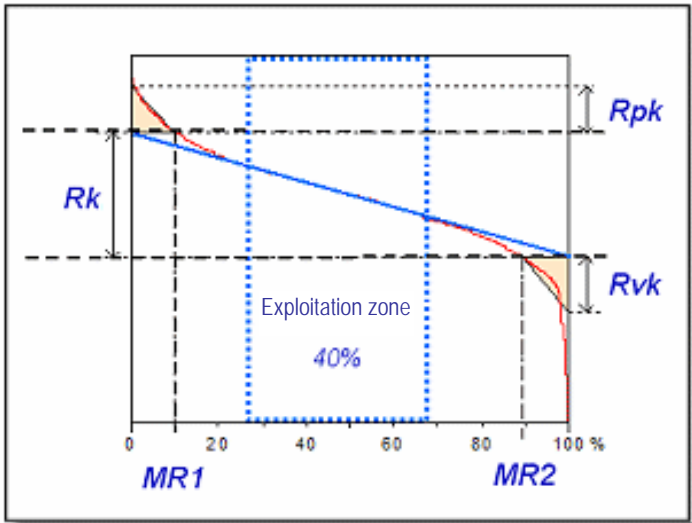
$$x = c_0 + c_1 z + \dots + c_9 z^9 , \quad (8)$$

but also there is not reached the required precision to take cutting conditions on account, not only machining method and achieved roughness. It is obvious that it is difficult to prescribe the shape of the Abbott curve in its entirety.

Three-segment Abbott curve approximation

The Abbott curve approximation problem is how many coefficients should be prescribed. Useful is three-segment Abbott curve approximation. The basic idea is to establish a procedure where the Abbott curve is expressed by three straight lines, this means that the roughness profile is divided into three areas. The basis of this technique is determined using the Abbott curve interpolation. In the solutions we get 2 points on a line j and then we obtain 3 lines, which we can describe more easily. Line between the points (x_2, z_2) and (x_3, z_3) is referred as exploitation line. Abbott curve shape would be enough to prescribe in this exploitation zone. Anchor zone for the machining is unavailable.

Through the middle part of the Abbott curve is layed a line and its intersection with the axis of ordinates determines the characteristic distance from the start of Abbott curve, which give us its one significant parameter. The second parameter is the horizontal distance (parallel to the x axis) of the intersection from Abbott curve. The same can be done at the bottom of Abbott curve, which gives us 4 parameters to its description.



- R_{pk} – reduced profile height,
- R_k – functional (exploitational) part of the profile,
- R_{vk} - reduced depth of profile depression,
- M_{r1} – the smallest material share of the basic section,
- M_{r2} – the largest material share of the basic section.

Fig. 6. Three-segment Abbott curve approximation according to standard STN ISO 13 565-2 describable by parameters $R_k, R_{pk}, R_{vk}, M_{r1}, M_{r2}$ [4]

One-segment approximation of Abbott curve – slope of curve

Three-segment approximation of Abbott curve can be changed to one-segment, if we approximate only its exploitation part, and we would be interested in particular section between 30% and 40% of the surface profile (position of cut profile).

Also important are angles $\alpha_0, \alpha_1, \alpha_2$, in them particular angle α_0 of line j with central profile line m (Fig. 4). When prescribing the shape of the Abbott curve in its exploitation zone, it could be only for instance inclination angle of the standard Abbott curve in

exploitation zone to axis of abscissa, or even on the y-axis from the beginning of exploitation zone (in agreed proportions, e.g. per cent from above).

Angle α_0 mostly represents character of material share curve of profile and it could be also prescribed for functional surface area by designer. [8]

The resulting equation of an approximated line in the exploitation zone can be write as the relation:

$$x = z \cdot \cot g\alpha_0 - z_1 \cdot \cot g\alpha_0. \quad (9)$$

Conclusion

The contribution brings the proposal to find a suitable mathematical expression of the shape of the Abbott curve and it is attempt to achieve multiparametrical quality determination of machined surface. Certainly there are not mentioned all the options, such as fractal dimension and others. We have shown that learning about Abbott curves contributes to a better understanding of machined surface quality in terms of theoretical - technology, metrology, and also for practical use.

References:

- [1] BÁTORA, B., VASILKO, K. *Obrobené povrchy*. Trenčín: Trenčianska univerzita, 2000, ISBN 80-88914-19-1.
- [2] BUDA, J., BÉKÉS, J. *Teoretické základy obrábania kovov*. Bratislava: Slovenské vydavateľstvo technickej literatúry, 1967.
- [3] BUMBÁLEK, B., ODVODY, V., OŠTÁDAL, B. *Drsnost povrchu*. Praha: SNTL – Nakladatelství technické literatury, 1989.
- [4] <http://www.digitalmetrology.com/Papers/CylinderBoreNoBkgd.pdf>, Dostupné na internete: 8.12.2008
- [5] JANÁČ, A., BÁTORA, B., BARÁNEK, I., LIPA, Z. *Technológia obrábania*. Bratislava: Vydavateľstvo STU, 2003. ISBN 80-227-2031-3
- [6] JANÁČ, A., LIPA, Z., PETERKA, J. *Teória obrábania*. Bratislava: Vydavateľstvo STU, 2006. ISBN 80-227-2347-9
- [7] Katalóg meracej techniky HOMMELWERKE, Kolbenova 159/7, 190 00 Praha: 2004.
- [8] LIPA, Z., TOMANÍČEK, S., TOMANÍČKOVÁ, D. Príspevok k trojúsečkovej aproximácii Abbottovej krivky. In *CO-MAT-TECH 2002, 10. medzinárodná vedecká konferencia*. Trnava: MtF STU, 2002, pp. 327-332.
- [9] LIPA, Z. Abbottove krivky sústružených povrchov. In *Medzinárodná konferencia STROJNÉ INŽINIERSTVO 2001*. Bratislava: Sjf STU, 2001. pp. 317-319.
- [10] OSANNA, P. H., JANÁČ, A., PROSTREDNIK, D. Oberflächenbewertung – Abbottkurve, ein neues Verfahren. In *Zborník medzinár. ved. konf. CO-MAT-TECH '96*. Trnava, 1996, pp. 275-280.
- [11] STN EN ISO 4287:1999: *Geometrické špecifikácie výrobkov (GPS). Charakter povrchu: Profilová metóda – Termíny, definície a parametre charakteru povrchu*.