MATHEMATICAL DESCRIPTION OF MATERIAL FLOW

György KOVÁCS, Peter KOŠŤÁL

Ing. György Kovács, PhD., University of Miskolc, Faculty of Mechanical Engineering and Informatics, Institute of Logistics, Miskolc-Egyetemvaros, 3515 Miskolc, Hungary, e-mail: altkovac@uni-miskolc.hu
doc. Ing. Peter Košťál, PhD., Slovak University of Technology, Faculty of Materials Science and Technology, Institute of Manufacturing Technologies, ul. J. Bottu 25, 91724 Trnava, Slovakia, e-mail: peter.kostal@stuba.sk

Abstract

Optimal design of logistical processes is essential for profitable operation or simulation of supply chains and supply chain members. The precise mathematical description of material flows between objects (which could be companies, machines etc.) is basic information for the logistical process design. In this paper the typical types of material flows will be introduced.

Key words

logistics, material flow, simulation

INTRODUCTION

A logistics system includes the material flow, equipment flow, information flow, energy flow, value flow, money flow and resource flow inside logistical processes of procurement, production/service, distribution and recycling.

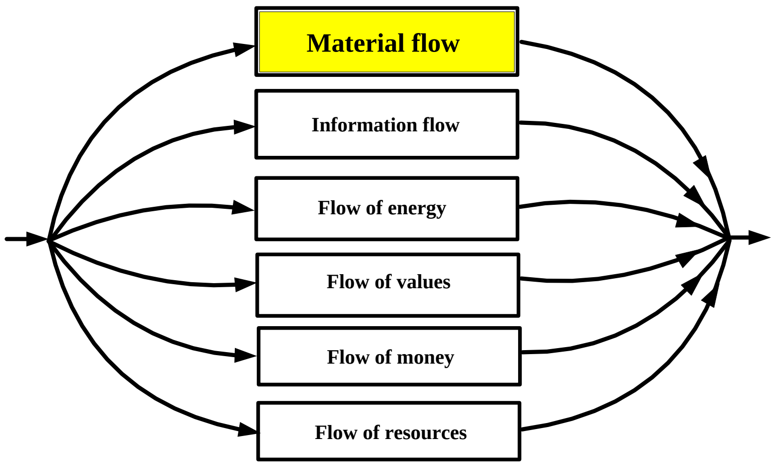


Fig. 1. *Definition of the logistics concept*

Material flow has a main role in the logistics system, as can be seen in the figure, so the planning of a material flow system is an important part of logistical and manufacturing process design.
Material flow includes the flow of raw materials, components, semi-finished products and final products over both a short and long distance.

Facilitating the flow of different raw materials, components, semi-finished products and final products to fulfil production and supply demands. Material flow includes horizontal and vertical delivery and additional activities like loading, unloading, etc.

CLASSIFICATION OF MATERIAL FLOW ACTIVITIES

Material flow activities can be classified into the following types:

- internal material flow (on the site of the company, between plants, between plants and warehouses, within technological processes, between machines) which is completed through the application of material flow machines and equipment;
- external material flow (generally transportation over a long distance out of the company site) which is carried out by transport vehicles by road, rail, inland water, see, etc.

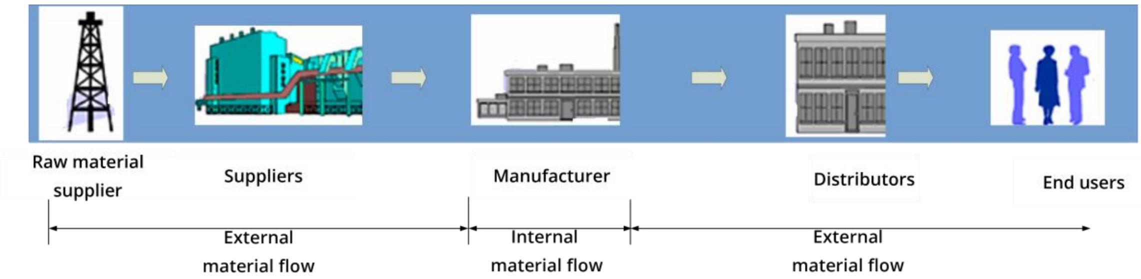


Fig. 2. *External and internal material flows*

DESIGN OF MATERIAL FLOW SYSTEMS

Important steps of the material flow system design are the following:

- Definition of required material flow activity and its requirements,
- **Mathematical description of the material flow,**
- Installation and location of plants, warehouses and machines,
- Loading unit formation, selection of optimal loading unit forming equipment,
- Selection of optimal material flow machines,
- Determination of the required number of material flow machines,
- Determination of the required capacity demand for storage and warehousing,
- Controlling the material flow system,
- Simulation and evaluation of the material flow system.

In this paper the main focus is on the description of the material flow.

MATHEMATICAL DESCRIPTION OF MATERIAL FLOW

Material flow intensity

Material flow intensity is the material flow per unit time, which can be considered in different dimensions:

- mass flow intensity:

$$q_m = \frac{\Delta M}{\Delta t}\left[\frac{m^3}{min}\right];\left[\frac{m^3}{hour}\right],$$

where: ΔM is the mass flow per Δt time period, volume flow intensity:

$$q_V = \frac{\Delta V}{\Delta t}\left[\frac{m^3}{min}\right];\left[\frac{m^3}{hour}\right],$$

where: ΔV is the volume flow per Δt time period.

- piece flow intensity:

$$q_n = \frac{\Delta n}{\Delta t}\left[\frac{pieces}{min}\right];\left[\frac{pieces}{hour}\right],$$

where: Δn is the piece flow per Δt time period.

Material flow efficiency

Material flow efficiency is the multiplication of traffic data and distance data.

Material flow (traffic) matrix:

$$\underline{Q} = \begin{matrix} 1 \\ \vdots \\ i \\ \vdots \\ m \end{matrix}\overset{1\cdots j\cdots n}{\begin{bmatrix} & & \\ & q_{i,j} & \\ & & \end{bmatrix}}$$

- m: is the number of sources
- n: is the number of destinations
- q_{ij}: is the quantity of material flow from the i-th object to the j-th object [pieces, ton, pieces of loading units, etc.]

Distance matrix:

Material flow effectiveness matrix:

Material flow efficiency is the multiplication of material flow quantity data and distance data. Minimization of the material flow efficiency is a very often used improvement aim in practice.

$$\underline{W} = \underline{L} \cdot \underline{Q}$$

$$\underline{W} = \begin{matrix} 1 \\ \vdots \\ i \\ \vdots \\ m \end{matrix}\overset{1\cdots j\cdots n}{\begin{bmatrix} & & \\ & l_{i,j} \times q_{i,j} & \\ & & \end{bmatrix}}$$

- m: is the number of sources
- n: is the number of destinations
- $q_{ij} \cdot l_{ij}$: is the material flow efficiency realized by material flow between the i-th object and j-th object [piece·m, ton·km, etc.]

The total material flow work of the system can be obtained by summarizing the elements of columns and rows of the $\underline{\underline{W}}$ matrix. The total material flow work can be reduced by reducing the quantity of material flow (q_{ij}) or reducing the distance of material flow (l_{ij}). The reduction of quantity is not possible in most cases, for example if the amount of products to be manufactured is given, the same amount of components is required for the assembly, and the number of components cannot be reduced. But in most cases the material flow distance (l_{ij}) can be reduced, for example by transportation of goods by a shorter way, or by redesigning the factory layout.

Material flow performance

Material flow performance is the multiplication of material flow intensity data and distance data.

$$\underline{P} = \underline{Q}^t \cdot \underline{L}$$

$$\underline{P} = \begin{matrix} 1 \\ \vdots \\ i \\ \vdots \\ m \end{matrix} \overset{1 \cdots j \cdots n}{\begin{bmatrix} & & \\ & q^t_{ij} \cdot l_{ij} & \\ & & \end{bmatrix}}$$

- *m:* is the number of sources
- *n:* is the number of destinations
- l_{ij}: is the distance between the *i*-th object and *j*-th object [m, km, etc.]
- q^t_{ij}: is the quantity of material flow intensity between the *i*-th object and *j*-th object [pieces/second, ton/hour, etc.]
- $q^t_{ij} \cdot l_{ij}$: is the material flow performance realized by material flow between the *i*-th object and *j*-th object [piece·m/sec, ton·km/hour, etc.]

CLASSIFICATION OF MATERIAL FLOWS – BASED ON BEHAVIOR OVER TIME

Material flows can be classified into two groups based on behavior over time.

Discrete material flow

In case of discrete material flow, the flow of goods is intermittent, not continuous. The operation times can be defined (periodical or non-periodical) in the description of material flow.

Continuous material flow

In the case of continuous material flow, the flow of goods is continuous, without interruption.

Material flow machines with discontinuous operation

Discontinuous machines have different operation cycles, for instance sometimes they transport goods, and sometimes they stop and wait for the next operation task. This machine group includes forklifts or cranes. These machines are generally mobile equipment.

Material flow machines with continuous operation

Continuous machines are always in motion, independently of whether they are transporting goods or not. This machine group includes belt conveyors or chain conveyors. These machines are generally non-mobile equipment.

CLASSIFICATION OF MATERIAL FLOWS – BASED ON PREDICTABILITY

Material flows can be classified into two groups based on predictability.

Deterministic material flow

The behavior and characteristics of this material flow are predictable. Modeling and mathematical description of deterministic processes is very simple (there are no random events in the processes).

Stochastic material flow

This process is also called random process, which means that the realization of the material flow is modified by randomly occurring events. Random events can be defined by probability variables. Most real material flow processes are stochastic, which means they are influenced by random events, e.g. traffic jams or machine breakdowns can occur in the course of a transport task.

ALTERNATIVE MATERIAL FLOW-TIME FUNCTIONS

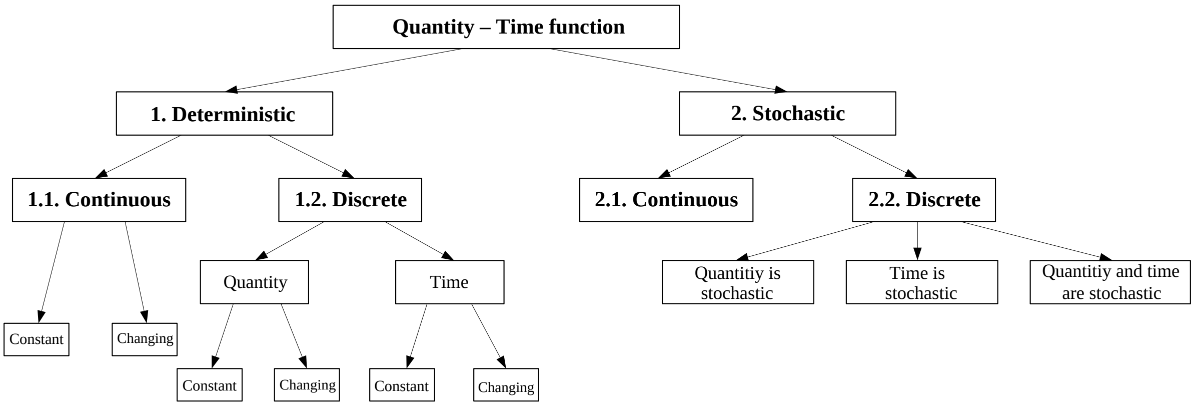
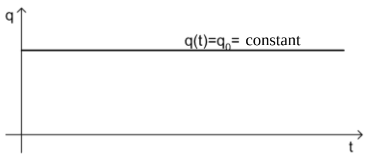


Fig. 3. *Alternative material flow-time functions*

The quantity of material flow (q_{ij}) between sources and destinations is given as a function of time (t) as input data for system design or simulation analysis. The quantity-time functions have to be defined for goods or a group of goods, transport paths and time intervals.
Next we will look at the basic variations of material flow types.

Deterministic and continuous material flows

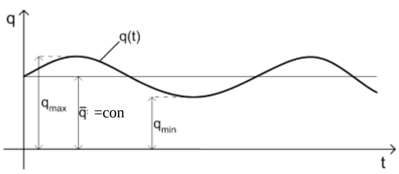
a) constant in time, stationary material flow



$q(t) = \bar{q} = \text{constant}$

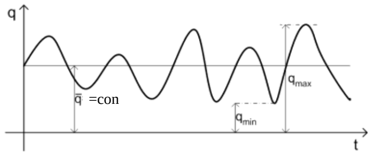
$\sigma_q = 0$

b) periodically changing, quasi-stationary material flow



$$\frac{\sigma_q}{\bar{q}} \leq a_0$$

c) non periodically changing, quasi-stationary material flow



$$\frac{\sigma_q}{\bar{q}} \le a_0$$

q: quantity of material flow

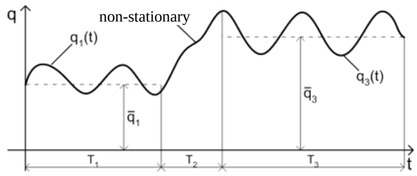
t: time

$\bar{q}$: average value of material flow quantity

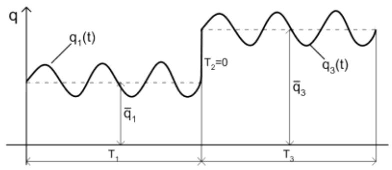
σ_q: deviation of average value of quantity-time function

a_0: limiting value (taking into consideration practical requirements)

d) changing quasi-stationary material flow with non-stationary sections

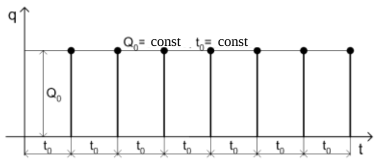


e) changing quasi-stationary material flow with high jumps

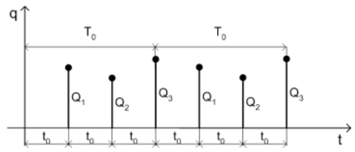


Deterministic and discrete material flows

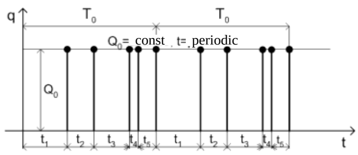
a) constant material flow, constant time intervals



b) changing (sometimes periodic) material flow, constant time intervals



c) constant material flow, changing (sometimes periodic) time intervals



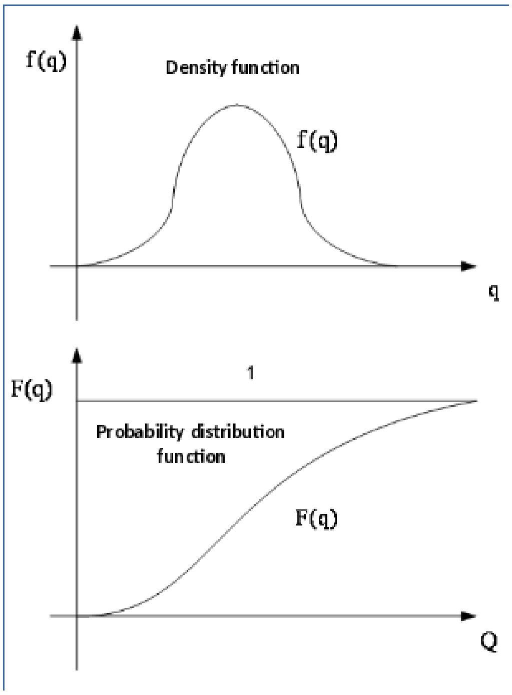
Stochastic material flows

Stochastic (random) events in the case of material flow processes can occur due to:

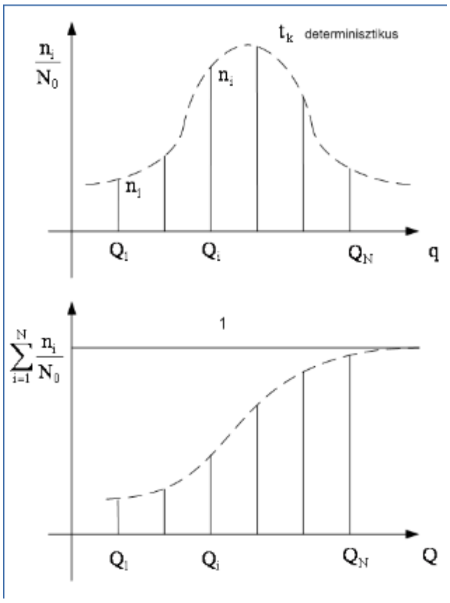
- changing customer demands, which results in the changing of production volume and product variety,
- operation problems of machines,
- breakdown of material flow machines,
- lack of equipment or human resources,
- lack of component supply (stocking and supplier problems).

Probability distribution functions and density functions are used for mathematical description of stochastic material flows. Gaussian distribution is the most often used function for simulation analysis of manufacturing processes.

Stochastic and continuous material flow



Stochastic and discrete material flow



Discrete material flow, time cycles are stochastic

CONCLUSION

Optimal design of logistical processes is essential for profitable operation of supply chains and supply chain members. The precise mathematical description of material flows between objects (which could be companies, machines, etc.) is basic information for the logistical process design.

The paper summarizes the most important terms relating to material flow and to the design process of material flow systems, focusing on mathematical description of the material flow. The calculation of most important parameters for material flow is detailed, e.g. for material flow efficiency and material flow performance. Classification of material flows are completed based on behavior in time and based on predictability. Finally the basic variations of material flow types are detailed.

Acknowledgement

This project has received funding from the European Union's Horizon 2020 research and innovation program under grant agreement No 691942".

References:

1. CSELÉNYI, J., ILLÉS, B. (ed.). 2006. *Design and control of material flow systems I.* (in Hungarian), book, University of Miskolc, ISBN 9636616728, Egyetemi Kiadó, p. 380.
2. PREZENSZKI, J. (ed.). 1997. *Logistics II.* (in Hungarian), book, Technical University of Budapest, ISBN 9630367408, p. 390.
3. BRAMEL, J., SIMCHI-LEVI D., 1997. *The Logic of Logistics: Theory, Algorithms, and Applications for Logistics Management*, ISBN 0-387-94921-6, Sringer-Verlag, p. 296.

4. LANGEVIN, A., RIOPEL, D. (ed.). 2005. *Logistics Systems: Design and Optimization***,** ISBN 0-387-24971-0, Sringer-Verlag, p. 401.
5. HUA, N. S., GUBÁN, M. 2014. *A data mining method for the solution of fluid-flow problem*. Advanced Logistic System: Theory and Practice, 7:(2), pp. 67-76.
6. GUBÁN, M., GUBÁN, Á. 2012. *Heuristic methods and examination of production scheduling*, Logistics - Eurasian Bridge: Materials of VIIth International scientific and practical conference. Krasnoyarsk: Centr Informacii, Conference proceeding, pp. 5-17. ISBN: 978-5-94617-265-3.

ORCID:
Peter Koštál 0000-0001-6622-6174